Center and Spread

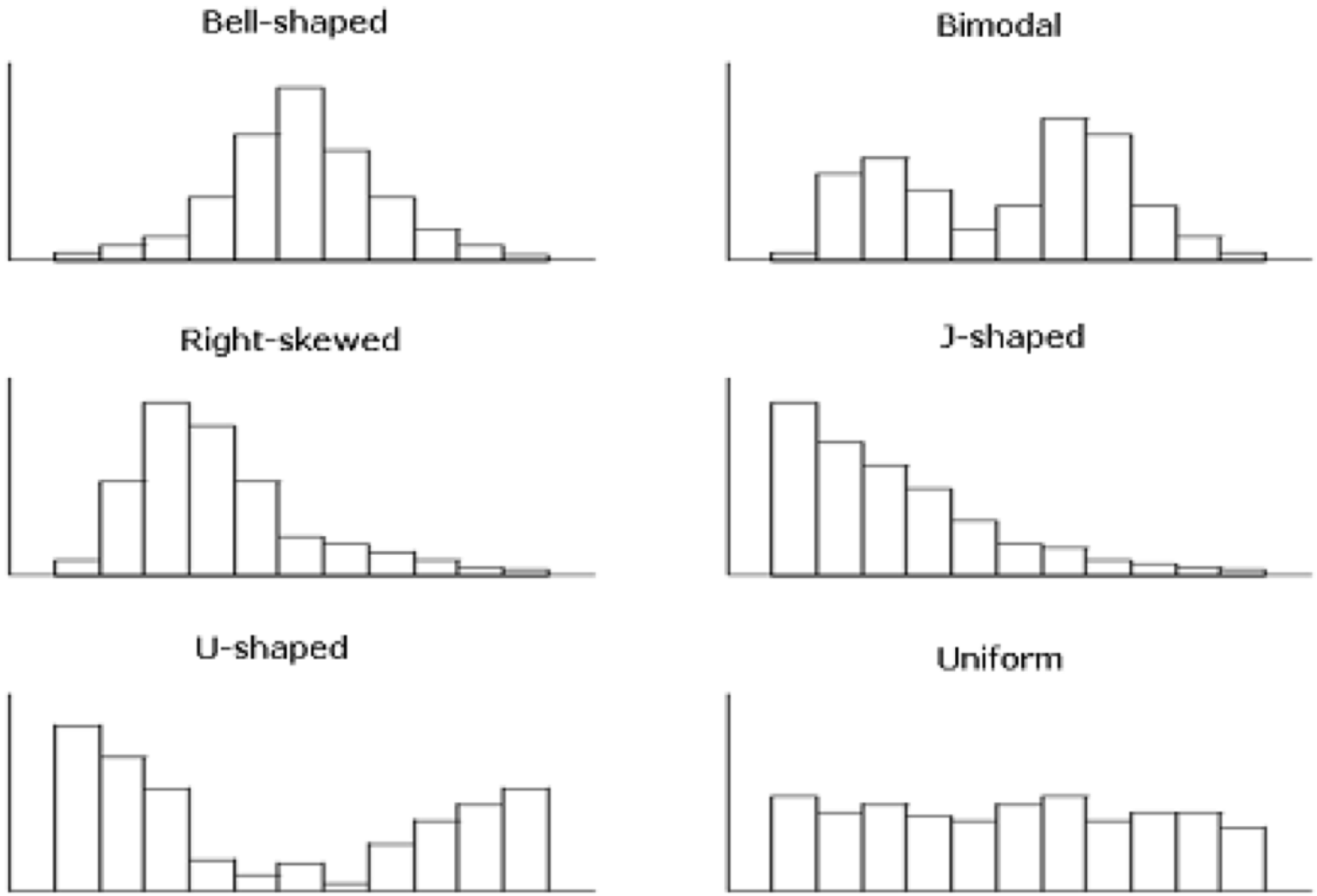
Cohen Chapter 3

EDUC/PSY 6600

"You can, for example, never foretell what any one man will do, but you can say with precision what an average number will be up to. *Individuals vary*, but percentages remain constant. So says the statistician."

-- Sherlock Holmes, *The Sign of Four*

Distributions Examples



Three Measures of Center

Mean

"Arithmetic Average" = add them all up and divide by the count

Not resistant: easily influenced by extreme values or outliers

Can do a "trimmed" mean (leave off the most extreme values, like 1% or 5%)

In a **POPULATION**: "Mu" (μ)

$$\mu = \frac{\sum X_i}{N}$$

In a **SAMPLE**: "X-bar" ($\bar{X}$) but APA uses "M" for abbreviation

$$\bar{X} = \frac{\sum X_i}{n}$$

Median

50th percentile, APA: "Mdn"

"Middle" value, when ordered/ranked in increasing order

ODD #: middle value

EVEN #: avg of 2 middle

Half the values are above, and half below

Easy for a computer to do

RESISTANT: NOT influenced by a few extreme values or outliers

Mode

Most common value, largest frequency, highest peak

Non-uniqueness - can have more than one mode

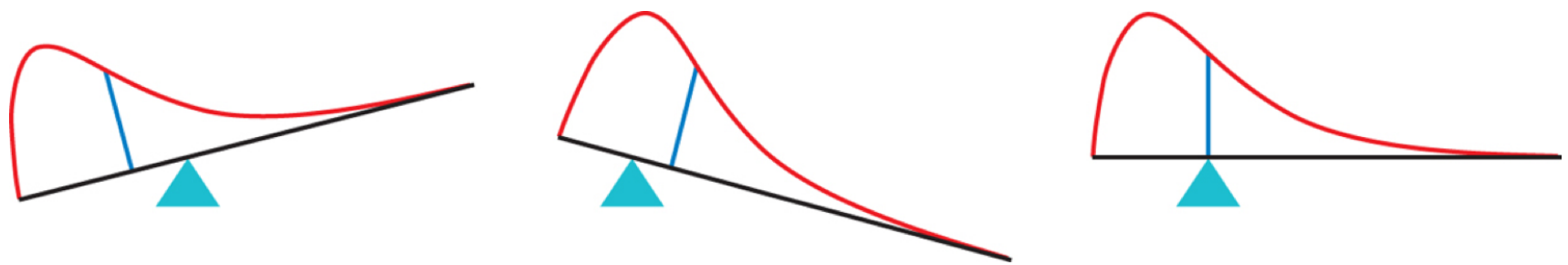
Doesn't always represent the 'center'

Do NOT usually use, other than descriptively

Mean vs. Median

Median: the center point, half of values are on each side, not affected by the skew, the "typical value"

Mean: the "balance" point, pulled to the side of the skew, not typical



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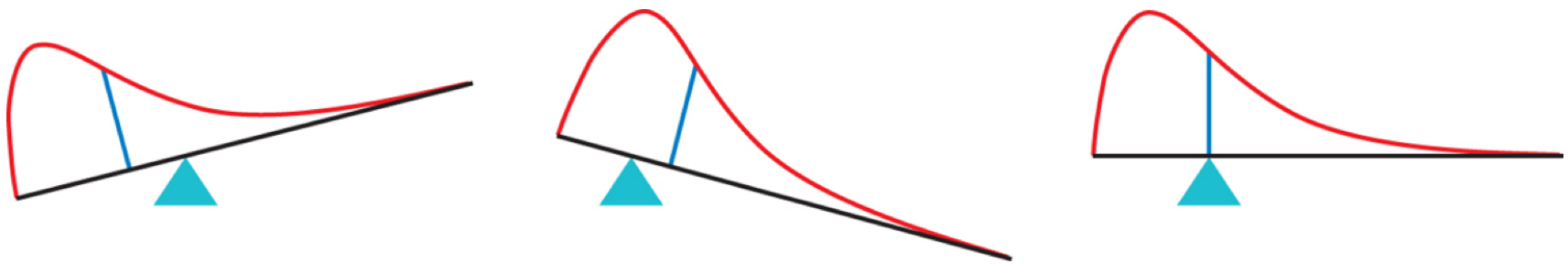
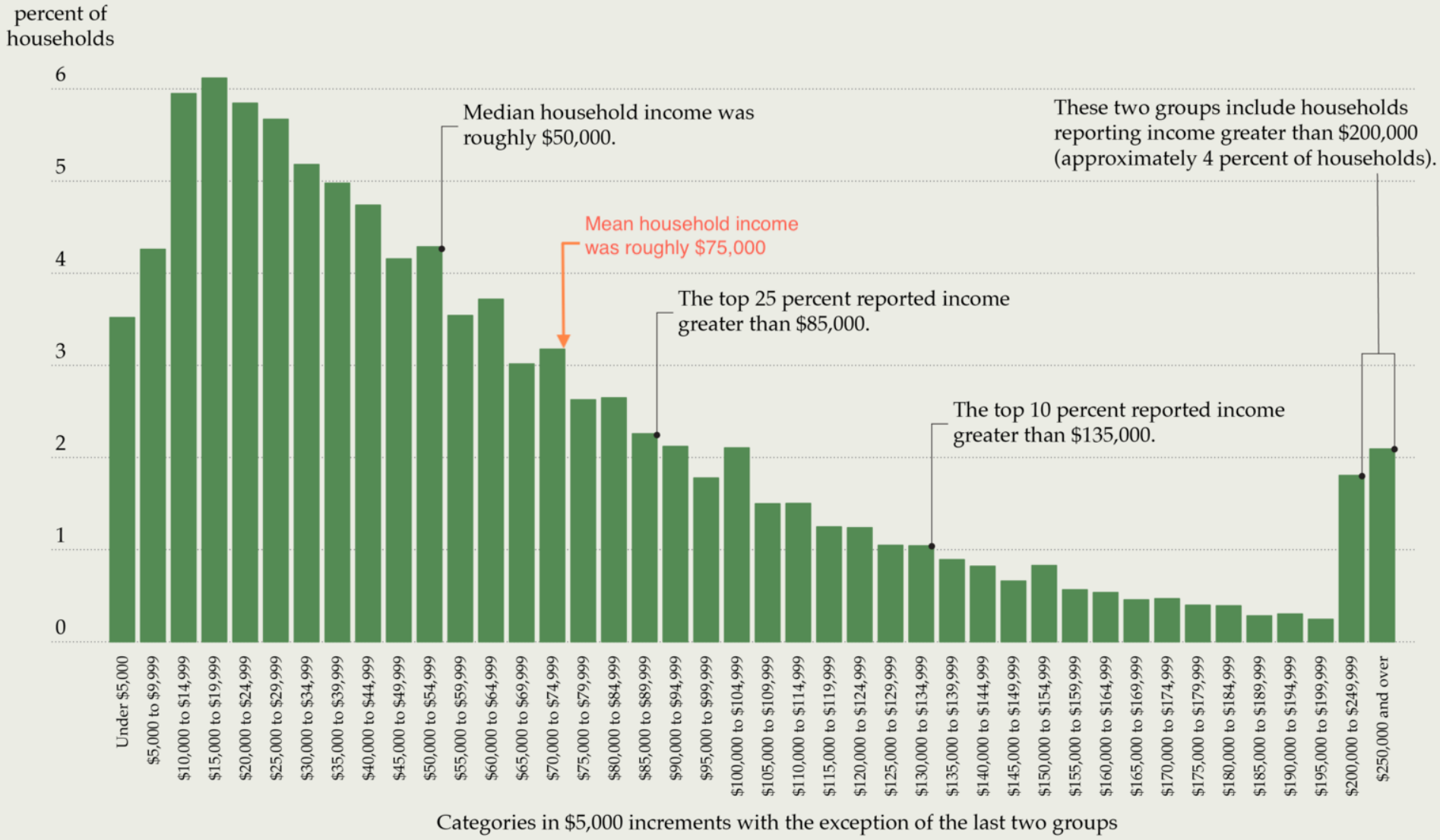


If distribution is symmetrical: mean = median

Distribution of annual household income in the United States

2010 estimate



Source: U.S. Census Bureau, Current Population Survey, 2011 Annual Social and Economic Supplement

Distributions and Numbers

- The MEDIAN is **resistant** & doesn't change much
- The MEAN is **influenced** & changes more!
- Average does NOT mean typical
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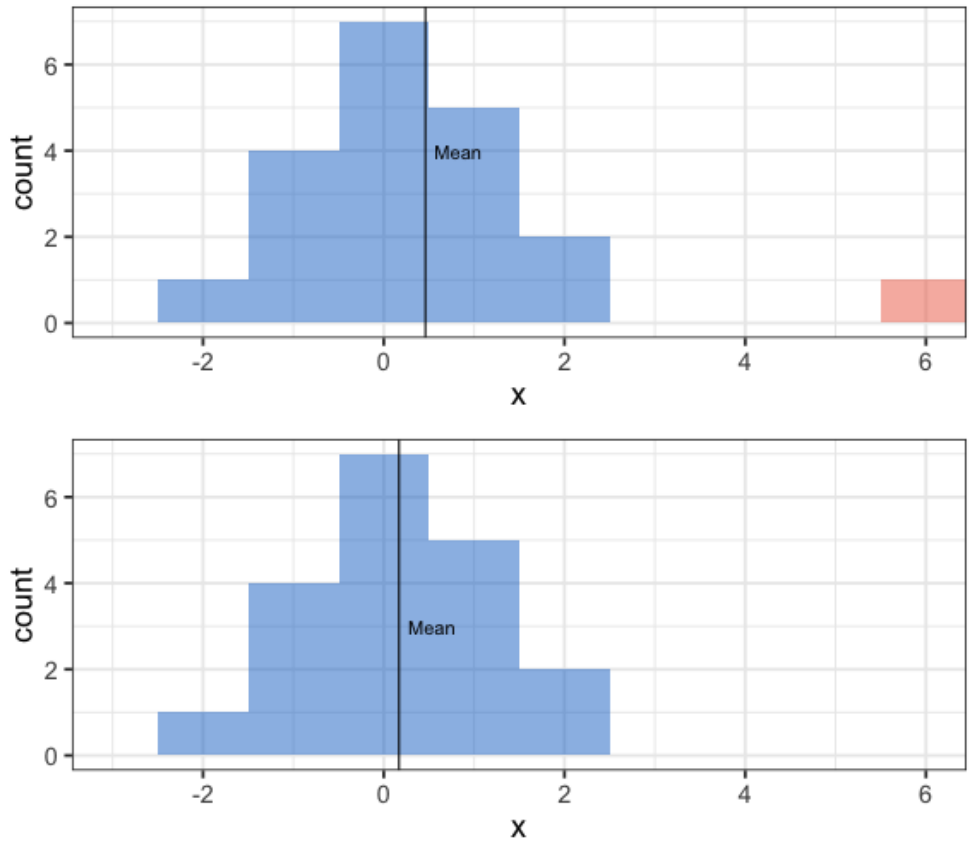

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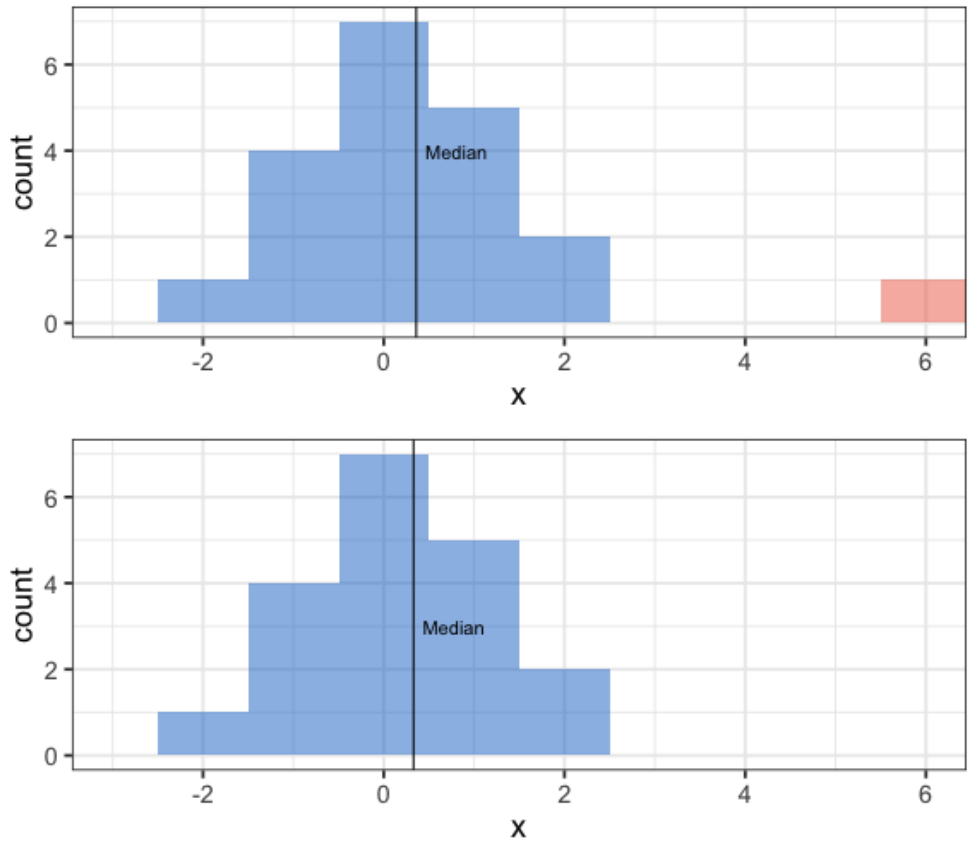

Three Measures of Spread

Range, IQR, & SIR

- Range = Max – Min
- Interquartile Range

IQR = Q3 – Q1

- Semi-Interquartile Range

SIR = (Q3 – Q1) / 2

- Range is super dependent on extreme values or outliers
- IRG & SIR more resistant

Variance

- DEVIANT: how far from the center (mean)
- SQUARE: so + & - don't cancel out to 0 (units are also squared)
- AVERAGE: summarize with a single value
- In a POPULATION: called "sigma-squared"

$$SS = \sum (X_i - mean)^2$$

$$MS = \frac{SS}{df}$$

$$\sigma^2 = \frac{\sum (X_i - \mu)}{N} = \frac{SS}{N} = MS$$

- In a SAMPLE: called "s-squared"

$$s^2 = \frac{\sum (X_i - \bar{X})}{n-1} = \frac{SS}{n-1} = \frac{SS}{df} = MS$$

- *Degrees of Freedom: df = n - 1*

Standard Deviation

- SQUARE-ROOT VARIANCE to get back to the original units
- In a POPULATION: called "sigma"

$$\sigma = \sqrt{\sigma^2} = \sqrt{\frac{\sum (X_i - \mu)}{N}} = \sqrt{\frac{SS}{N}} = \sqrt{MS}$$

- In a SAMPLE: called "s"

$$s = \sqrt{s^2} = \sqrt{\frac{\sum (X_i - \bar{X})}{n-1}} = \sqrt{\frac{SS}{n-1}} = \sqrt{MS}$$

Best Summary of the Data?

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Median and SIR

Skewed data or outliers

Mean and SD

Symmetrical and no outliers

Best Summary of the Data?

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Median and SIR

Skewed data or outliers

Mean and SD

Symmetrical and no outliers

A graph gives the best overall picture of a distribution

Properties of the Mean and SD

If you _____ the same CONSTANT number onto every score...	MEAN	STANDARD DEVIATION
ADD (or subtract)	ADD (or subtract) SAME amount	unchanged
MULTIPLY (or divide)	MULTIPLY (or divide) by the SAME amount	MULTIPLY (or divide) by the SAME amount

Skewness

- Degree of symmetry in distribution
- Can detect **visually** (histogram, boxplot)
- Skewness statistic
  - Based on cubed deviations from the mean
  - Divided by SE of skewness
  - $> \pm 2$ is a sign of skewed data

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$$Skewness = \frac{N}{N-2} \frac{\sum_{i=1}^{n}(X_i - \bar{X})^3}{(N-1)s^3}$$

- Interpreting skewness statistic
  - positive value = positive (right) skew
  - negative value = negative (left) skew
  - zero value = no skew

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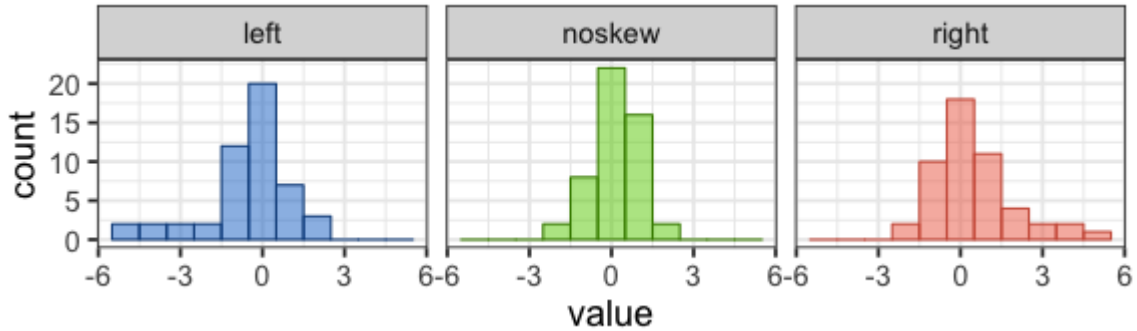

Kurtosis

$$Kurtosis = \frac{N(N+1)}{(N-2)(N-3)} \frac{\sum_{i=1}^{n}(X_i - \bar{X})^4}{(N-1)s^4} - 3\frac{(N-1)(N-1)}{(N-2)(N-3)}$$

- Degree of flatness in distribution
- Harder to detect visually
- Kurtosis statistic
  - Based on deviations from the mean (raised to 4th power)
  - Divided by SE of kurtosis
  - $> \pm 2$ is a sign of problems with kurtosis

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  - Based on deviations from the mean (raised to 4th power)
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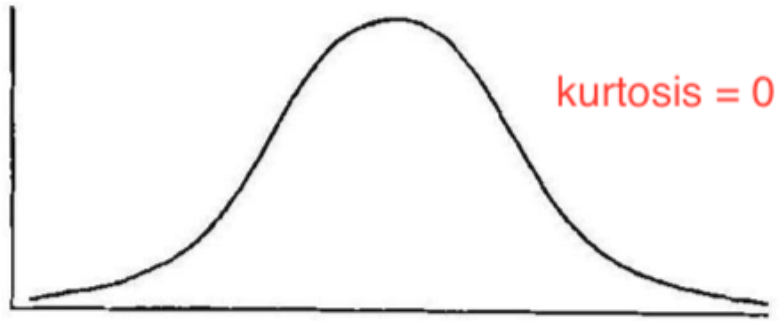
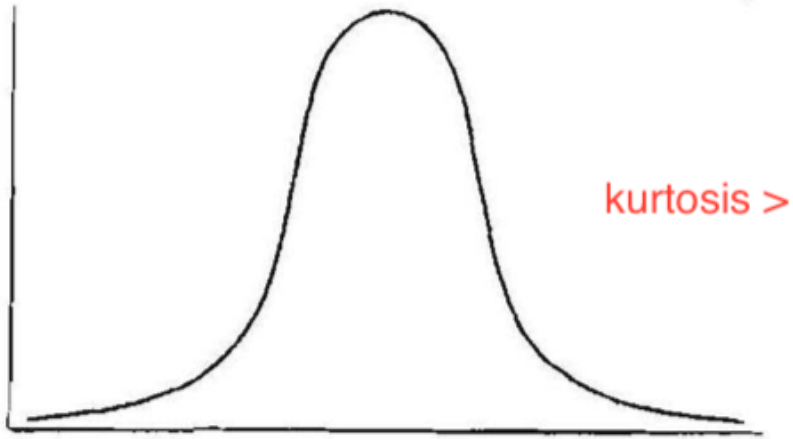
  - $> \pm 2$ is a sign of problems with kurtosis
- Interpreting kurtosis statistic
  - positive value = leptokurtic (peaked)
  - negative value = platykurtic (flat)
  - zero value = mesokurtic (normal)

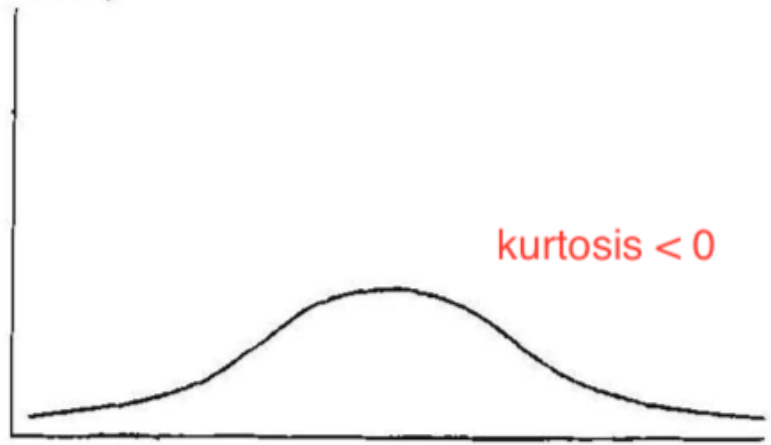
Kurtosis



Distribution H
(Mesokurtic distribution)



Distribution I
(Leptokurtic distribution)

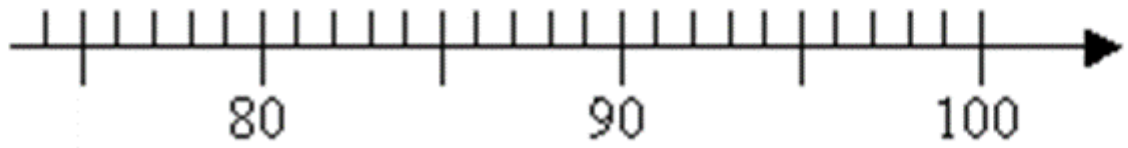


Distribution J
(Platykurtic distribution)

Five-Number Summary

Example of VERY small amount of data

77, 79, 80, 86, 87, 87, 94, 99



Five-Number Summary - Median

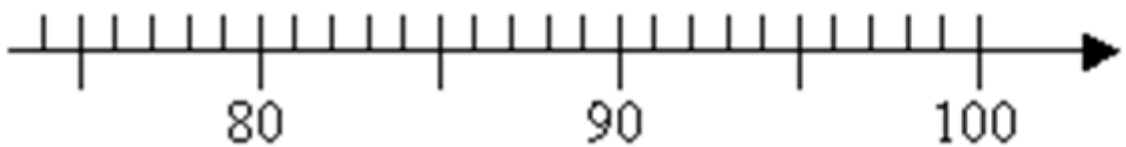
Example of VERY small amount of data

77, 79, 80, 86, 87, 87, 94, 99

Min = 77

M = 86.5

Max = 99



Five-Number Summary - Quartiles

Example of VERY small amount of data

77, 79, 80, 86, 87, 87, 94, 99

Min = 77

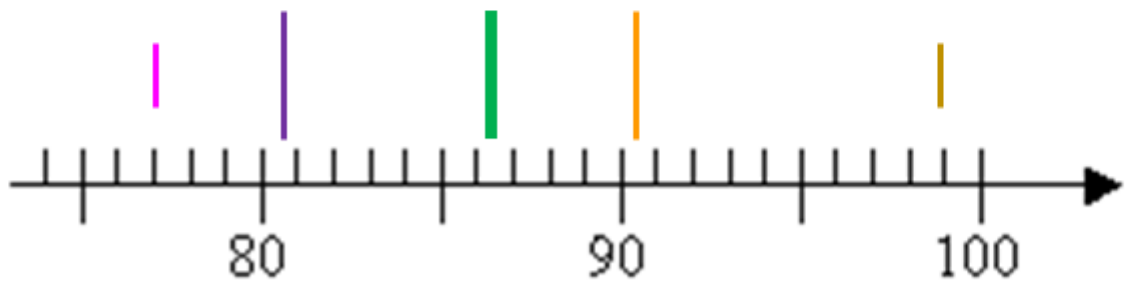
Q1 = 79.5

M = 86.5

Q3 = 90.5

Max = 99

Five-number summary = 77, 79.5, 86.5, 90.5, 99



Box-Plot = connect lines plotted above

Boxplots (Modified) - Lines

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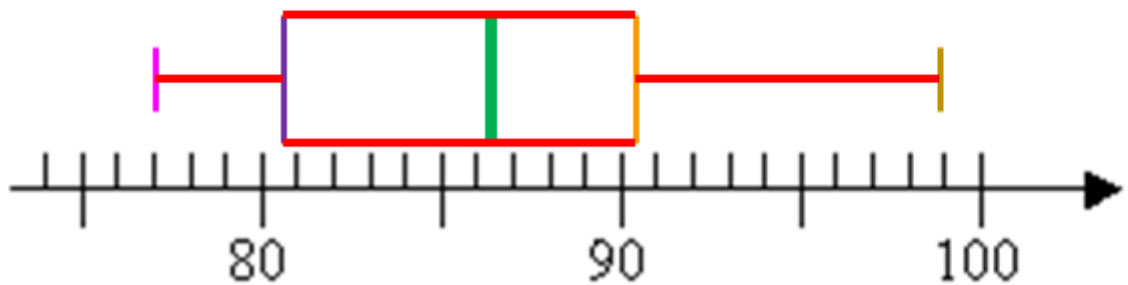

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Boxplots (Modified) - IQR and SIQR

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77, 79, 80, 86, 87, 87, 94, 99

Min = 77

Q1 = 79.5

M = 86.5

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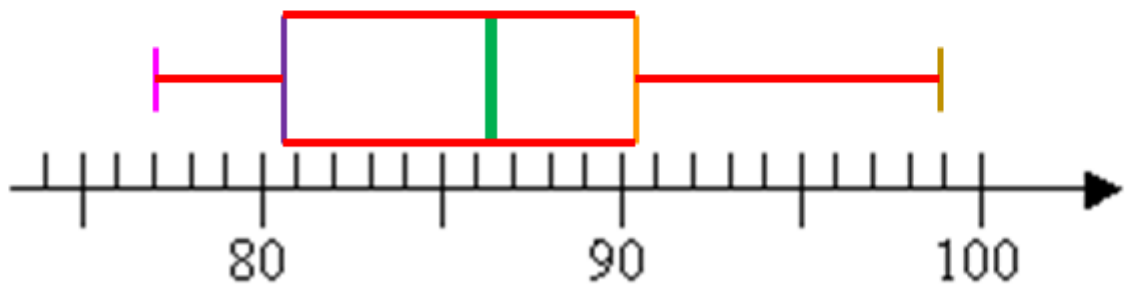
Five-number summary = 77, 79.5, 86.5, 90.5, 99

IQR = 90.5 – 79.5 = 11

SIQR = 11 / 2 = 5.5



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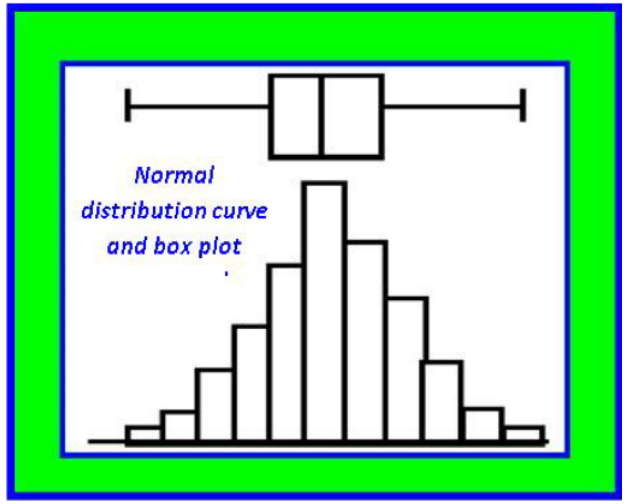
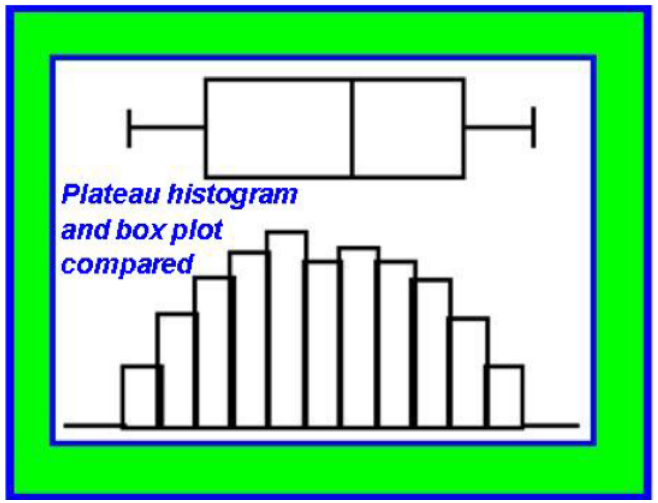
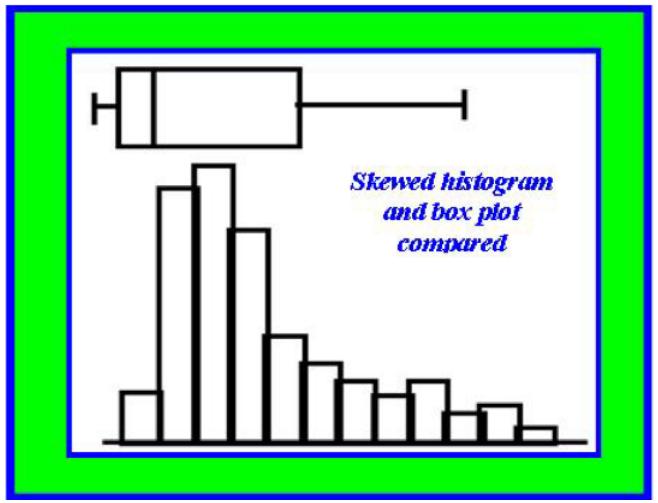
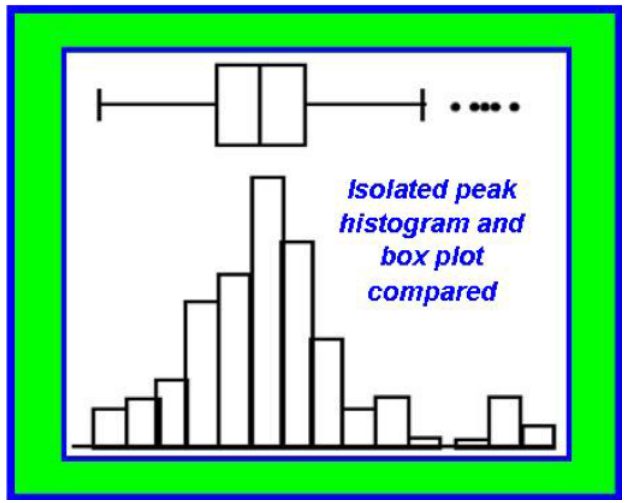
Boxplot vs. Histogram

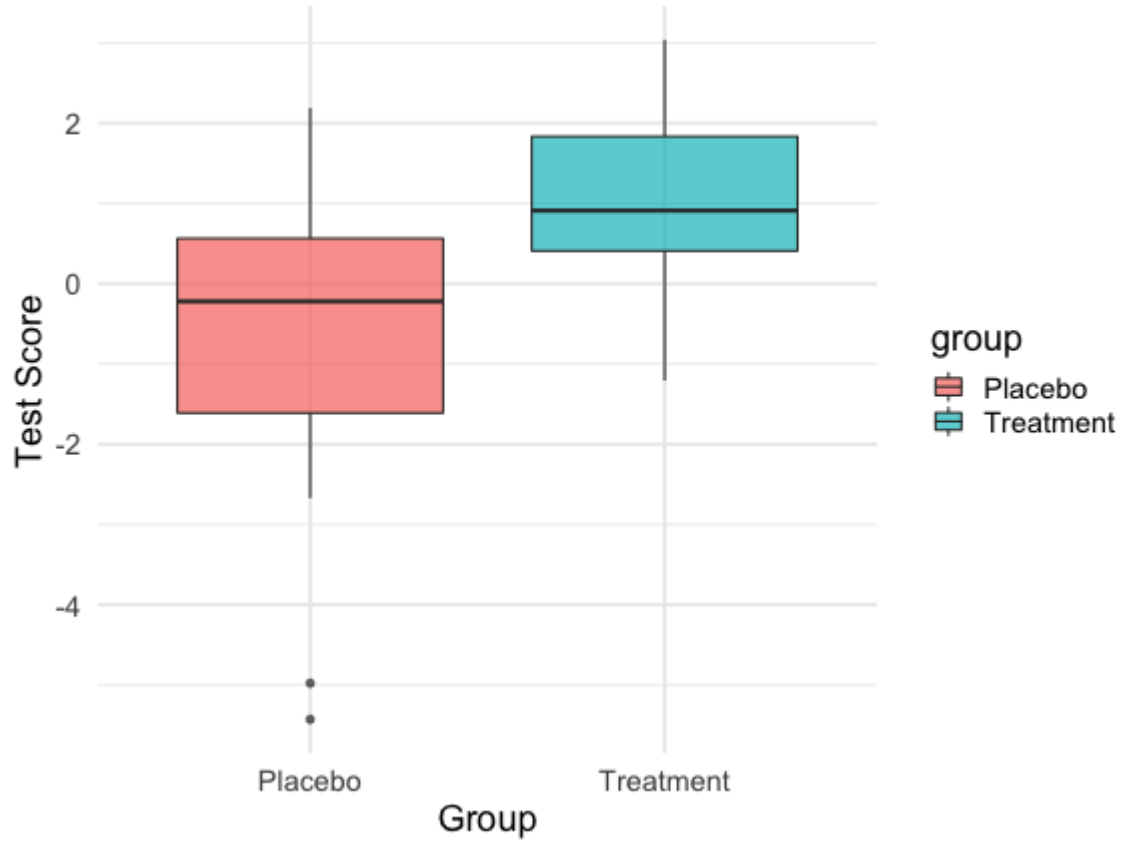




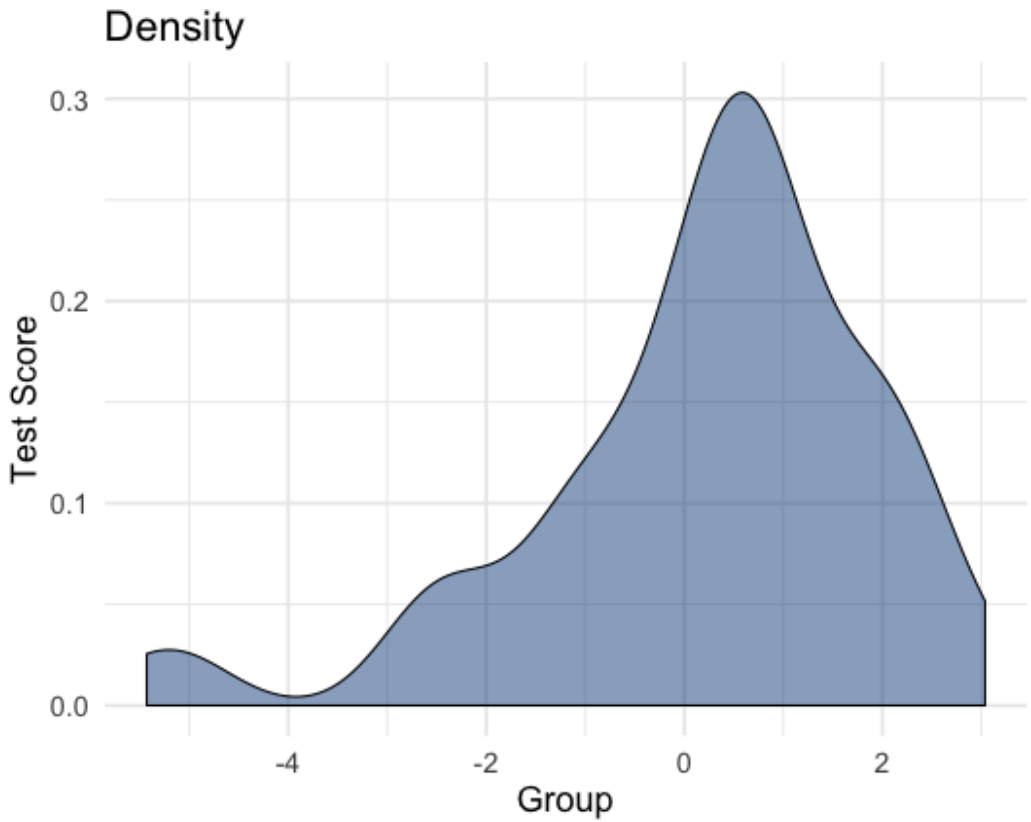


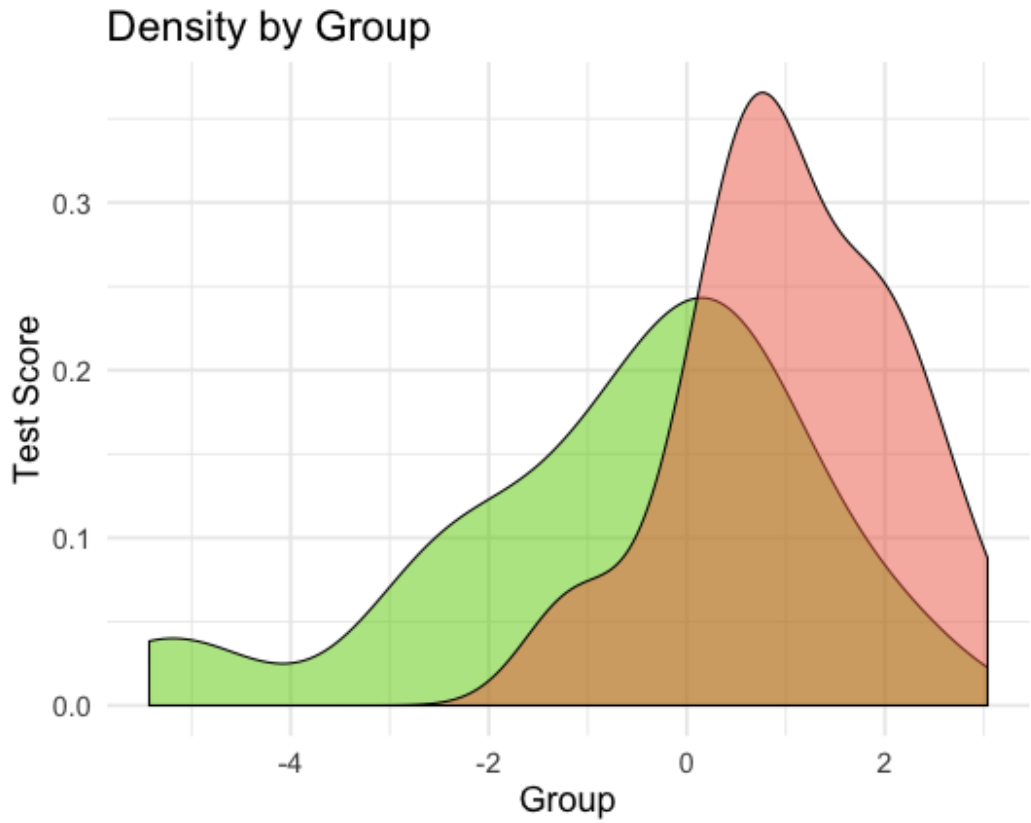


Boxplots by Group

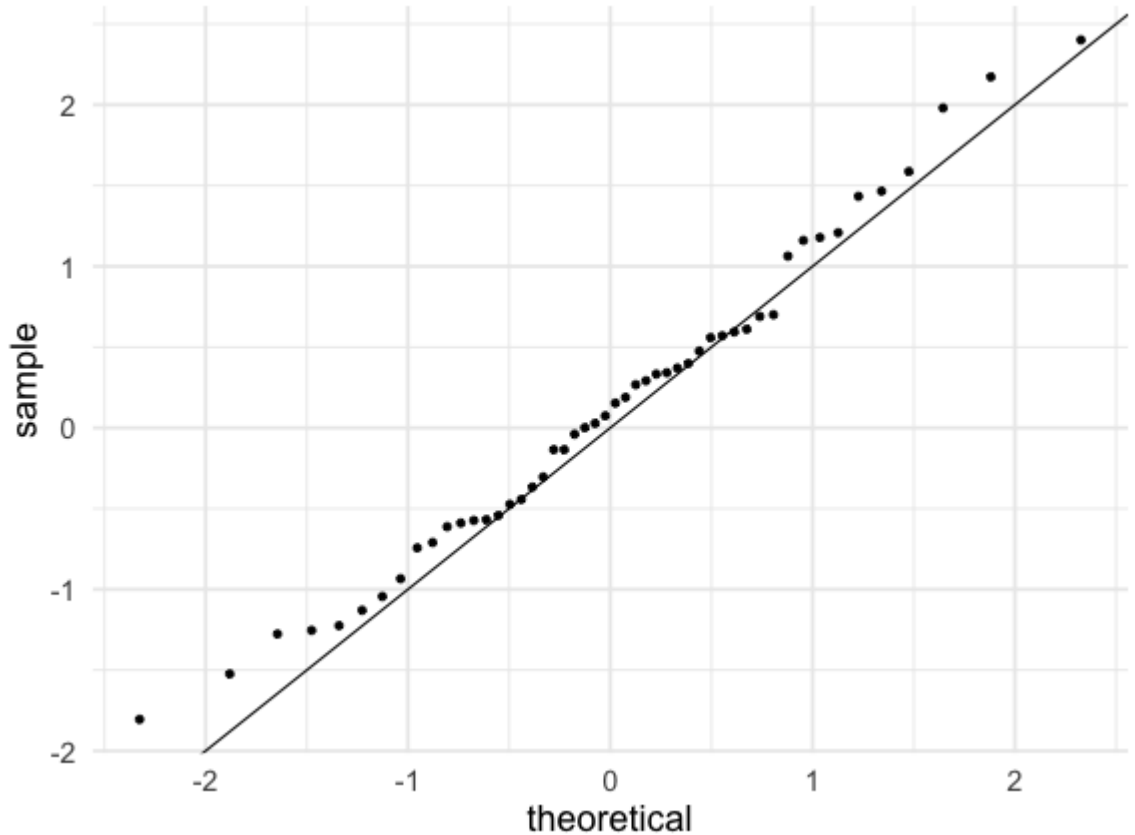


Density Plots





Quantile-Quantile (Q-Q) Plot



Let's Apply This To the Cancer Dataset (on Canvas)

Read in the Data

```
library(tidyverse)     # Loads several very helpful 'tidy' packages
library(rio)           # Read in SPSS datasets
library(furniture)     # Nice tables (by our own Tyson Barrett)
library(psych)         # Lots of nice tid-bits
```

```
cancer_raw <- rio::import("cancer.sav")
```

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```
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```

And Clean It

```
cancer_clean <- cancer_raw %>%
  dplyr::rename_all(tolower) %>%
  dplyr::mutate(id = factor(id)) %>%
  dplyr::mutate(trt = factor(trt,
                             labels = c("Placebo",
                                        "Aloe Juice"))) %>%
  dplyr::mutate(stage = factor(stage))
```

Frequency Tables with `furniture::tableF()`

```
cancer_clean %>%
  furniture::tableF(age, n = 8)
```

age	Freq	CumFreq	Percent	CumPerc
27	1	1	4.00%	4.00%
42	1	2	4.00%	8.00%
44	1	3	4.00%	12.00%
46	2	5	8.00%	20.00%
...	...	...	...	...
68	1	20	4.00%	80.00%
69	1	21	4.00%	84.00%
73	1	22	4.00%	88.00%
77	2	24	8.00%	96.00%
86	1	25	4.00%	100.00%

```
cancer_clean %>%
  furniture::tableF(trt)
```

trt	Freq	CumFreq	Percent	CumPerc
Placebo	14	14	56.00%	56.00%
Aloe Juice	11	25	44.00%	100.00%

Extensive Descriptive Stats `psych:describe()`

```
cancer_clean %>%
  dplyr::select(age, weighin, totalcin, totalcw2, totalcw4, totalcw6) %>%
  psych::describe()
```

```
         vars  n   mean    sd median trimmed   mad min   max range  skew
age         1 25  59.64 12.93   60.0   59.95 11.86  27  86.0  59.0 -0.31
weighin     2 25 178.28 31.98  172.8  176.57 21.05 124 261.4 137.4  0.73
totalcin    3 25   6.52  1.53    6.0    6.33  0.00   4  12.0   8.0  1.80
totalcw2    4 25   8.28  2.54    8.0    8.10  2.97   4  16.0  12.0  1.01
totalcw4    5 25  10.36  3.47   10.0   10.19  2.97   6  17.0  11.0  0.49
totalcw6    6 23   9.48  3.49    9.0    9.21  2.97   3  19.0  16.0  0.77
         kurtosis   se
age         -0.01 2.59
weighin      0.07 6.40
totalcin     4.30 0.31
totalcw2     1.14 0.51
totalcw4    -1.00 0.69
totalcw6     0.53 0.73
```

Smaller Set with `furniture::table1()`

For the Entire Sample

```
cancer_clean %>%
  furniture::table1(trt, age, weighin)
```

```
_____________________________________

              Mean/Count (SD/%)
              n = 25
trt
   Placebo    14 (56%)
   Aloe Juice 11 (44%)
age
              59.6 (12.9)
weighin
              178.3 (32.0)
_____________________________________
```

Breaking the Sample by a Factor

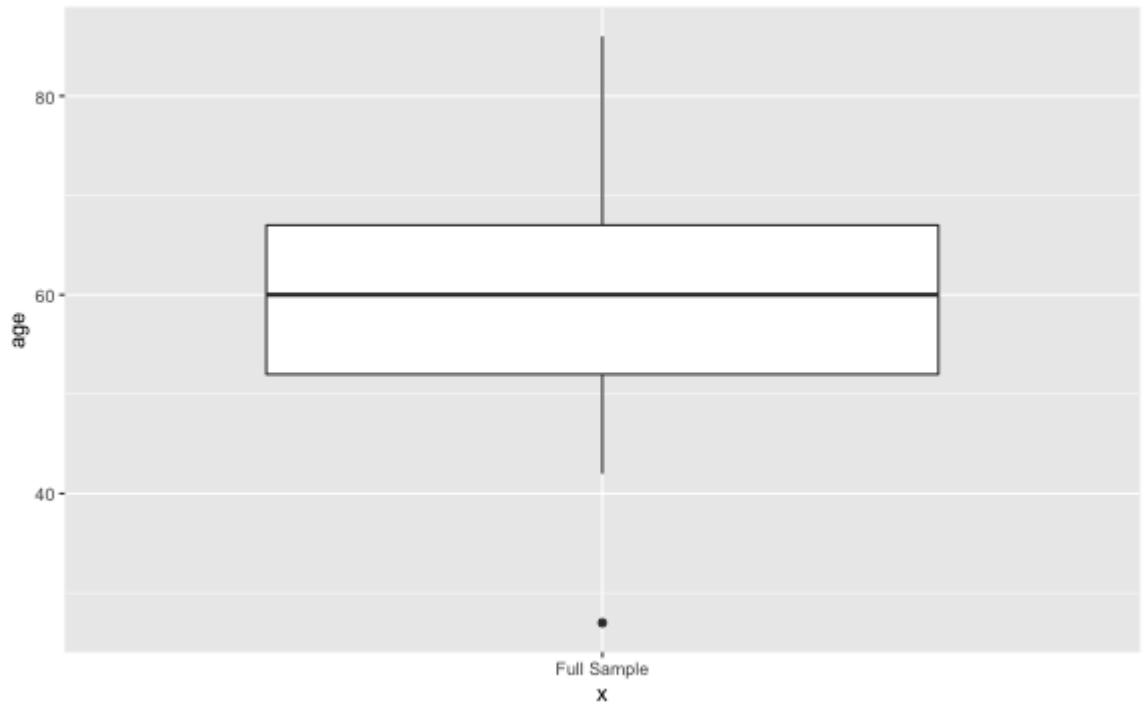
```
cancer_clean %>%
  dplyr::group_by(trt) %>%
  furniture::table1(age, weighin)
```

```
_____________________________________________

                    trt
          Placebo       Aloe Juice
          n = 14        n = 11
age
          59.8 (9.0)    59.5 (17.2)
weighin
          167.5 (23.0)  192.0 (37.4)
_____________________________________________
```

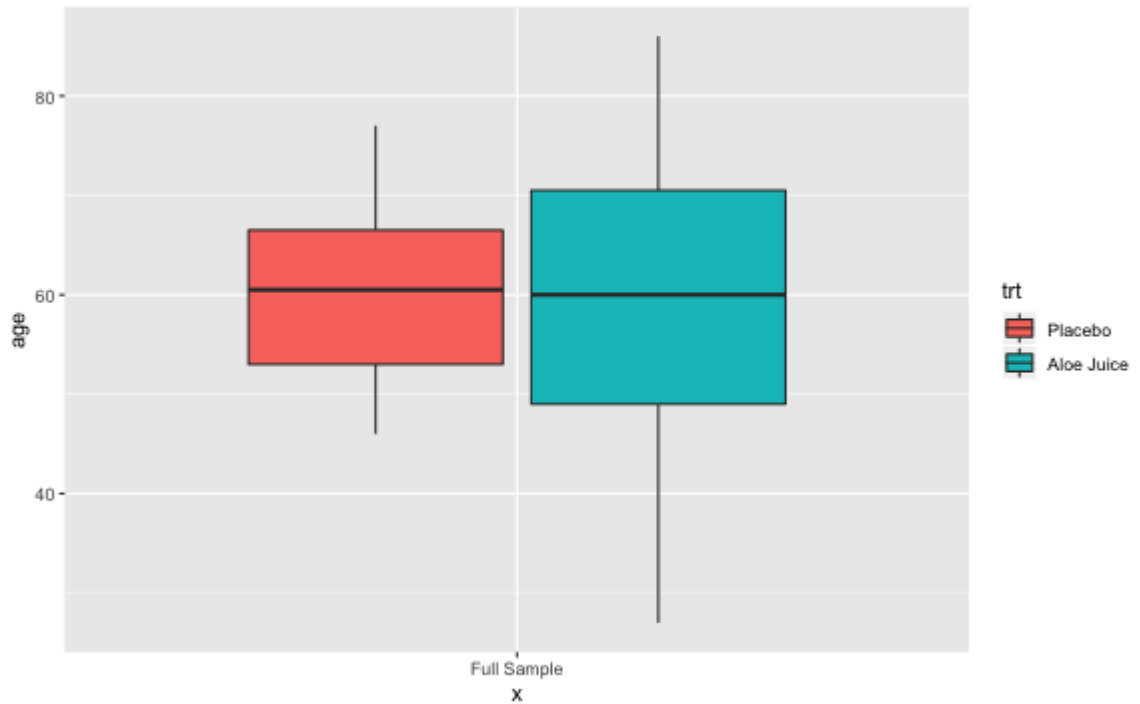
Boxplot, one one `geom_boxplot()`

```
cancer_clean %>%
  ggplot(aes(x = "Full Sample",    # x = "quoted text"
             y = age)) +           # y = contin_var (no quotes)
  geom_boxplot()
```



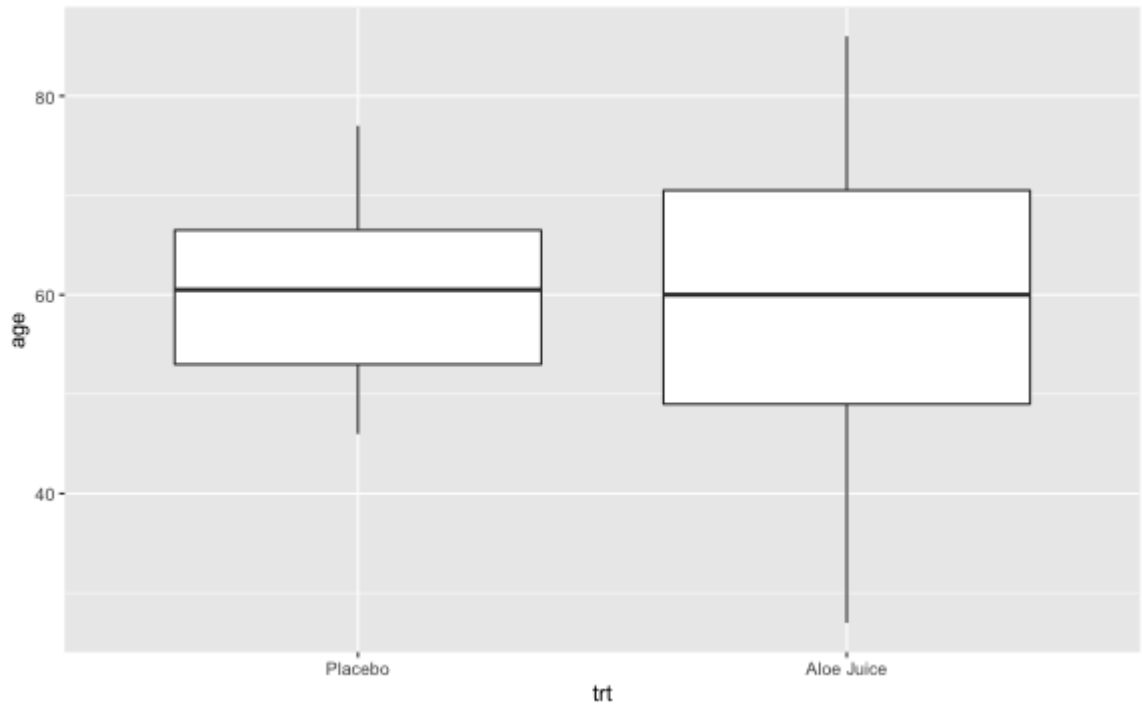
Boxplots, by groups - (1) fill color

```
cancer_clean %>%
  ggplot(aes(x = "Full Sample",          # x = "quoted text"
             y = age,                    # y = contin_var (no quotes)
             fill = trt)) +              # fill = group_var (no quotes)
  geom_boxplot()
```



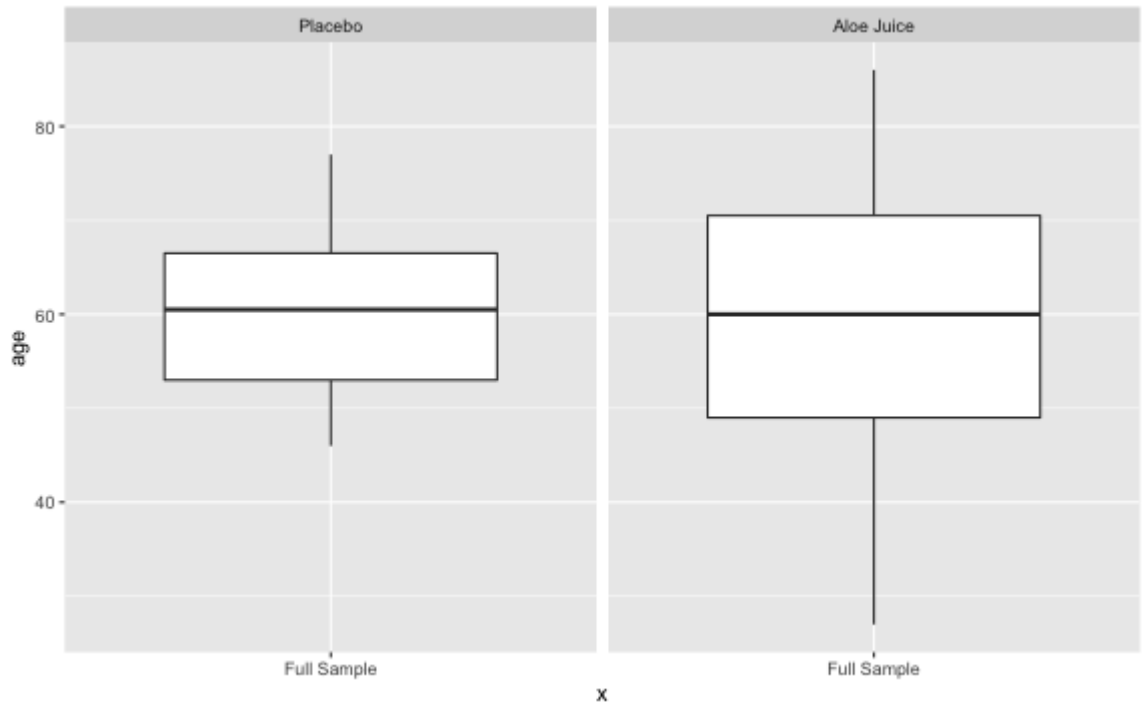
Boxplots, by groups - (2) x-axis breaks

```
cancer_clean %>%
  ggplot(aes(x = trt,              # x = group_var (no quotes)
             y = age)) +           # y = contin_var (no quotes)
  geom_boxplot()
```



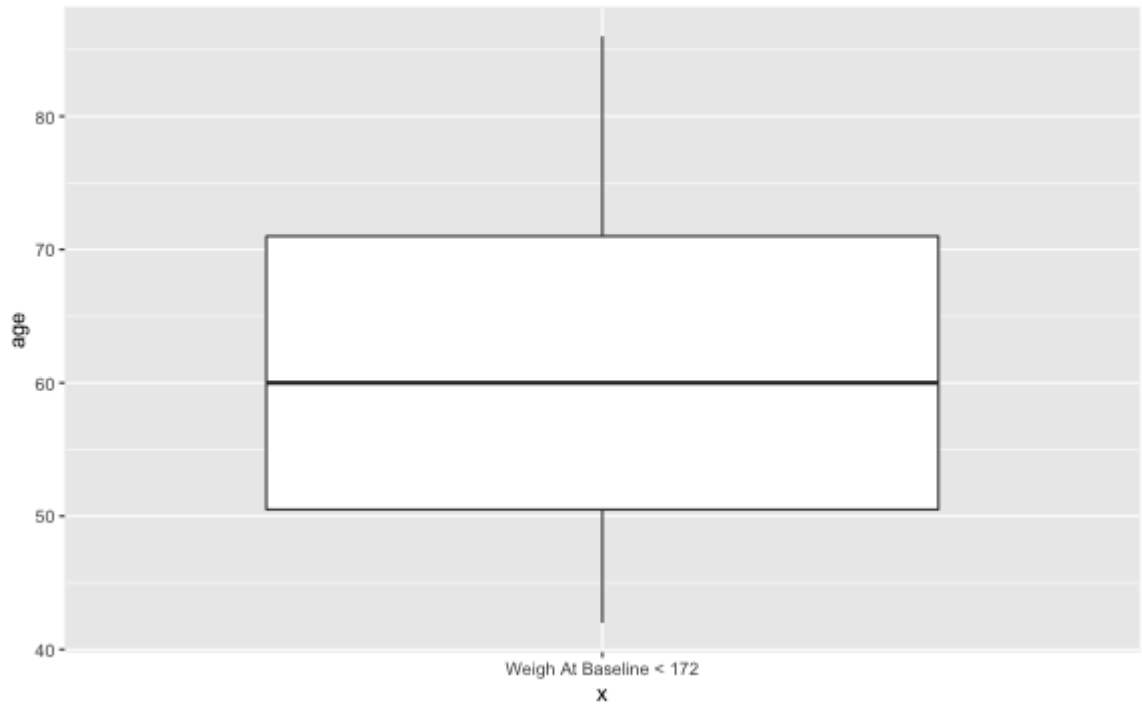
Boxplots, by groups - (3) seperate panels

```
cancer_clean %>%
  ggplot(aes(x = "Full Sample",    # x = "quoted text"
             y = age)) +           # y = contin_var (no quotes)
  geom_boxplot() +
  facet_grid(. ~ trt)              # . ~ group_var (no quotes)
```



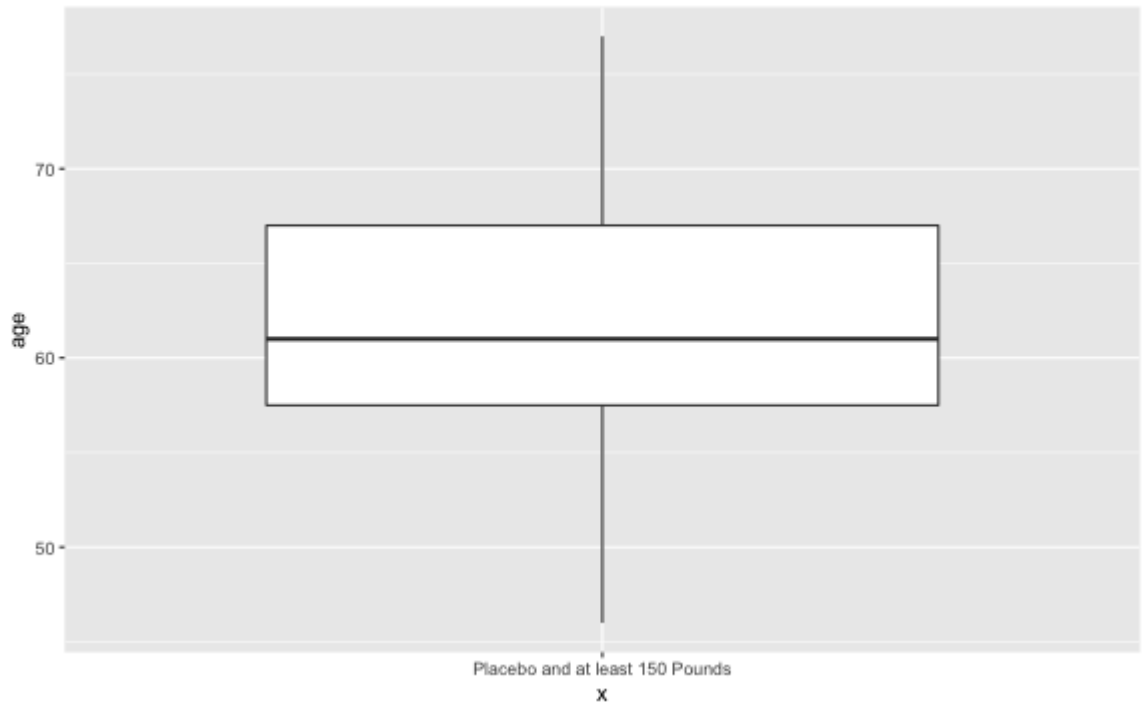
Boxplot for a Subset - 1 requirement

```
cancer_clean %>%                  # Less than 172 Pound at baseline
  dplyr::filter(weighin < 172) %>%
  ggplot(aes(x = "Weigh At Baseline < 172",
             y = age)) +
  geom_boxplot()
```



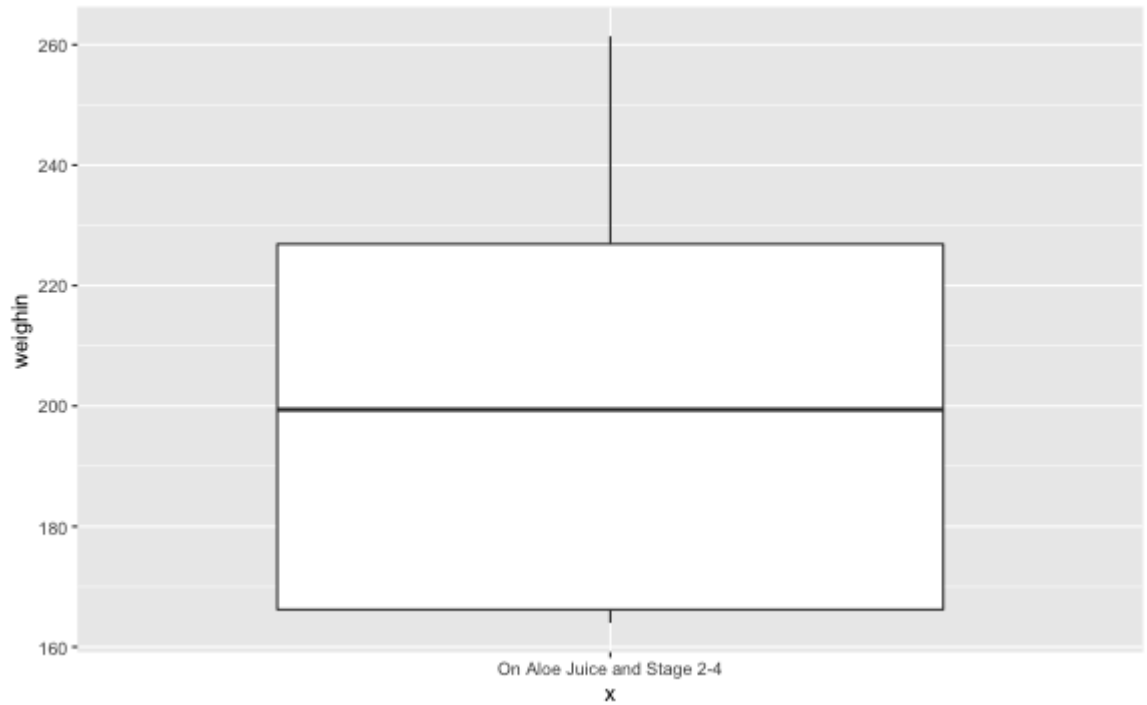
Boxplot for a Subset - 2 requirements

```
cancer_clean %>%            # At least 150 pounds AND not in Aloe group
  dplyr::filter(weighin >= 150 & trt == "Placebo") %>%
  ggplot(aes(x = "Placebo and at least 150 Pounds",
             y = age)) +
  geom_boxplot()
```



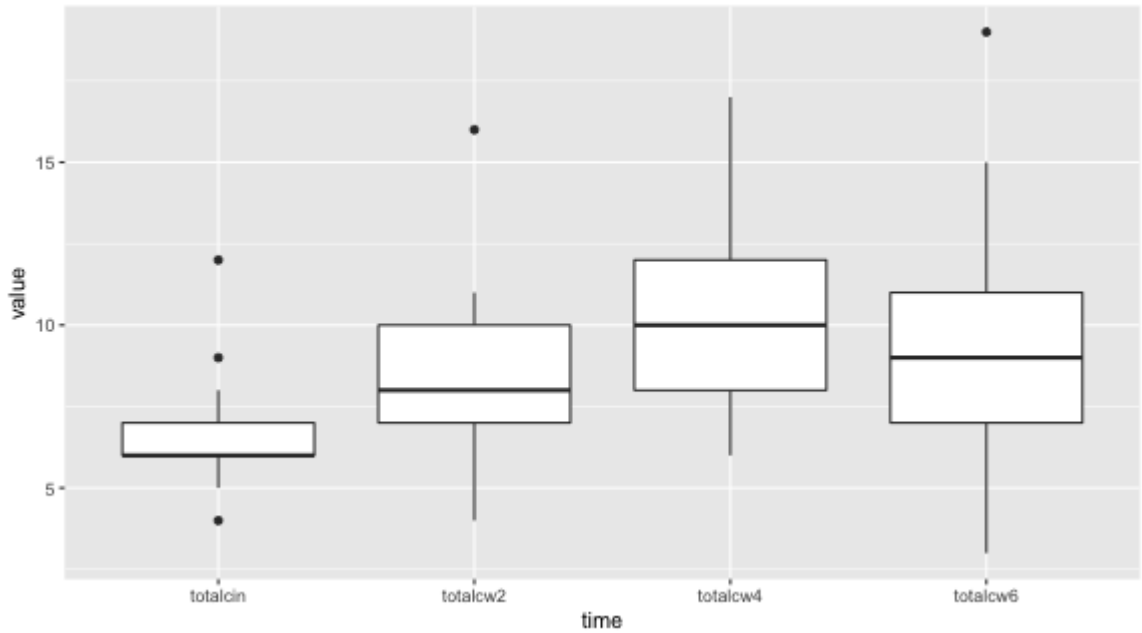
Boxplot for a Subset - 2 requirements (`%in%`)

```
cancer_clean %>%           # In Aloe group, but only stages 2-4
  dplyr::filter(trt == "Aloe Juice" & stage %in% c(2, 3, 4)) %>%
  ggplot(aes(x = "On Aloe Juice and Stage 2-4",
             y = weighin)) +
  geom_boxplot()
```



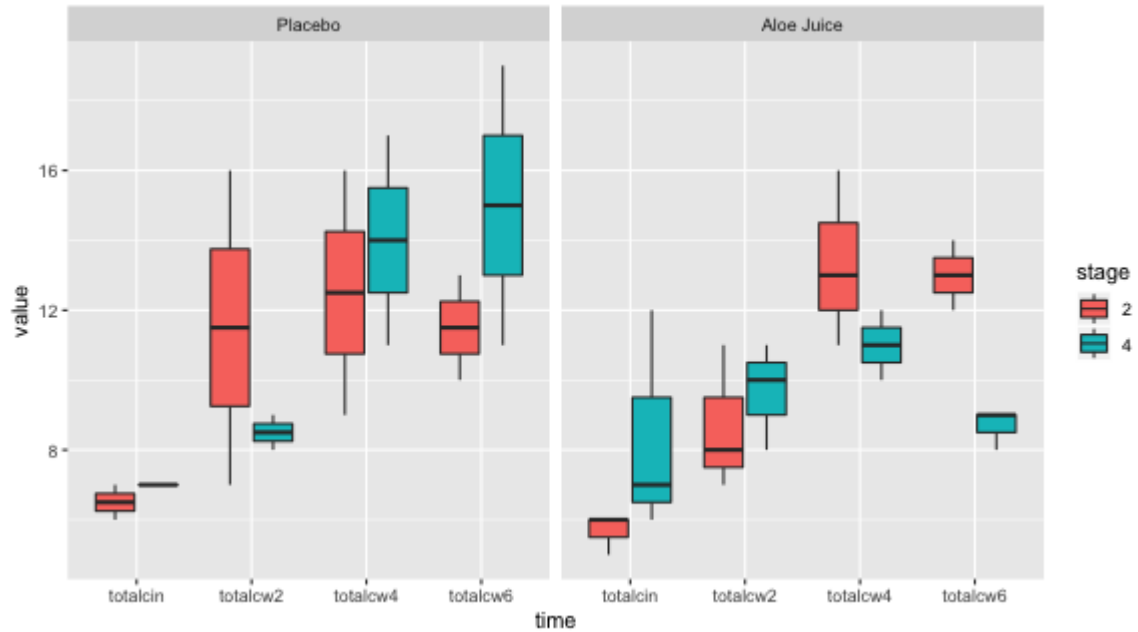
Boxplot for Repeated Measures

```
cancer_clean %>%
  tidyr::gather(key = "time",          # stack the repeated measures
                value = "value",
                totalcin, totalcw2, totalcw4, totalcw6) %>%
  ggplot(aes(x = time,
             y = value)) +
  geom_boxplot()
```



Boxplot: COMPLICATED!

```
cancer_clean %>%
  dplyr::filter(weighin > 130 & stage %in% c(2, 4)) %>%
  tidyr::gather(key = "time", value = "value",
                totalcin, totalcw2, totalcw4, totalcw6) %>%
  ggplot(aes(x = time, y = value, fill = stage)) +
  geom_boxplot() +
  facet_grid(. ~ trt)
```



Questions?

Next Topic

Standard and Normal