T-test for 2 Independent Sample Means

FOR EDUC/PSY 6600

"We cannot solve problems by using the same kind of thinking that we used when we created them."

Albert Einstein

Intro

- Same continuous DV compared across 2 independent (random) samples
- Is there a significant difference between the 2 group means?
  - Do 2 samples come from *different* normal distributions with the same mean?
- aka…
  - Independent-groups design
  - Between-subjects design
  - Between-groups design
  - Randomized-groups design

Steps of a Hypothesis test

$H_0: Diff_{\mu} = 0 \quad \mu_1 - \mu_2 = 0 \quad \mu_1 = \mu_2$

$H_1: Diff_{\mu} \neq 0 \quad \mu_1 - \mu_2 \neq 0 \quad \mu_1 \neq \mu_2$

"T-Test Statistic": $t = \frac{est - hyp}{SE_{est}}$

*even use z, if N > 100'ish

1) State the Hypotheses (Null & Alternative)
2) Select the Statistical Test & Significance Level
   - α level
   - One vs. Two tails
3) Select random samples and collect data
4) Find the region of Rejection
   - Based on α & # of tails
5) Calculate the Test Statistic
   - Examples include: z, t, F, χ^2
6) Make the Statistical Decision

Steps of a Hypothesis test

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Separate Variance t-Test

(need HOV)

$$t = \frac{(\overline{X_1} - \overline{X_2}) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1{}^2}{n_1} + \frac{s_2{}^2}{n_2}}}$$

$$\boldsymbol{df} = n_1 + n_2 - 2$$

Pooled-Variance t-Test

(different sample sizes)

$$t = \frac{(\overline{X_1} - \overline{X_2}) - (\mu_1 - \mu_2)}{\sqrt{s_p{}^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$min(n_1, n_2) - 1 < \boldsymbol{df} < n_1 + n_2 - 2$$

$$s^2 = \frac{ss}{n-1}$$

$$s_p{}^2 = \frac{SS_1 + SS_2}{n_1 + n_2 - 2}$$

Example 1

Group 1 - Drug	Group 2 - Placebo
11	11
1	11
0	5
2	8
0	4

- In order to assess the efficacy of a new antidepressant drug, 10 clinically depressed participants are randomly assigned to one of two groups. Five participants are assigned to Group 1, which is administered the antidepressant drug for 6 months. The other 5 participants are assigned to Group 2, which is administered a placebo during the same 6 month period
- Assume that prior to introducing the treatments, the experimenter confirmed that the level of depression in the 2 groups was equal
- After 6 months, all participants are rated by a psychiatrist (blind to participant assignment) on their level of depression

Example 1

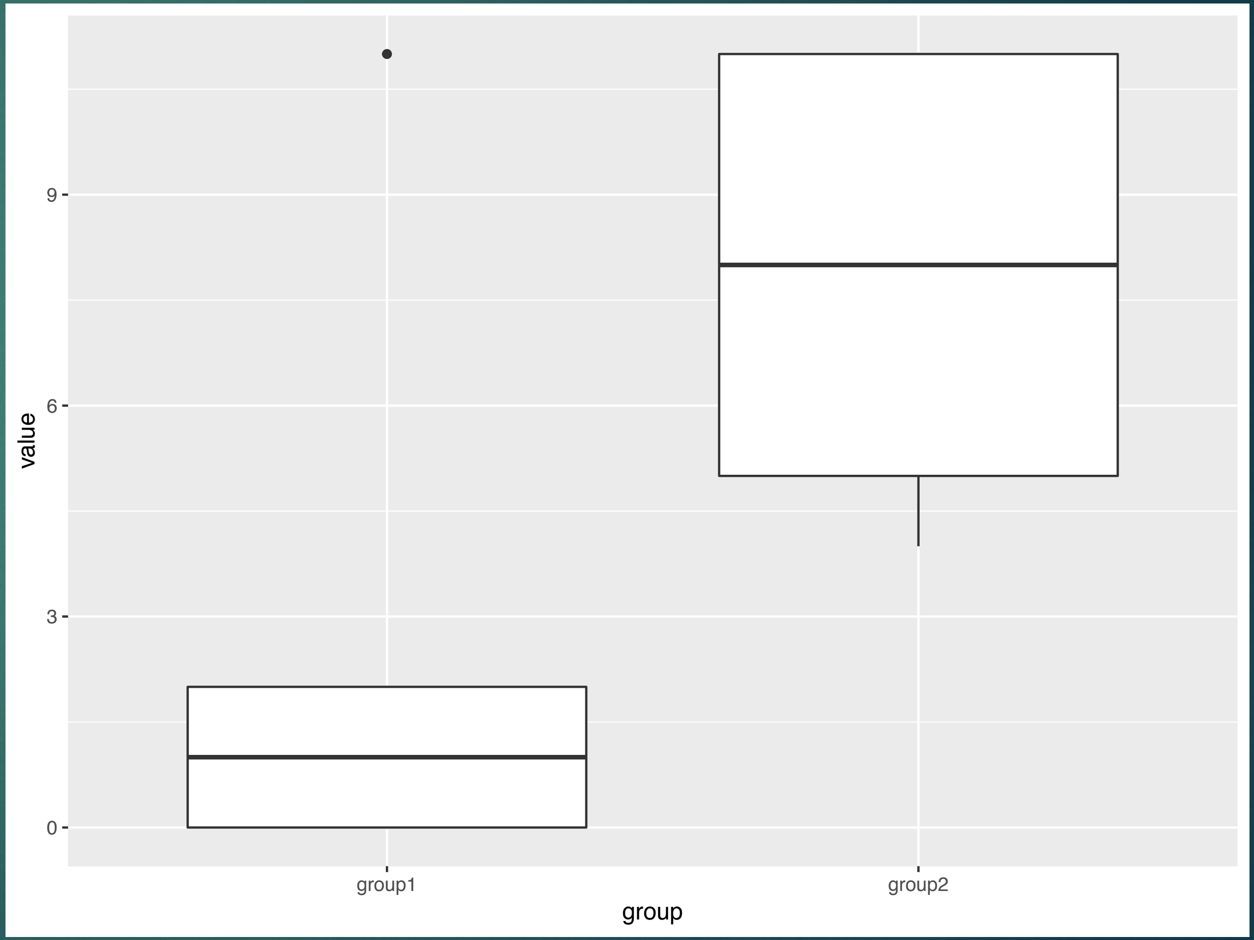
Group 1 - Drug	Group 2 - Placebo
11	11
1	11
0	5
2	8
0	4

```r
library(tidyverse)
library(furniture)

## Manually input data
df <- data.frame(group1 = c(11, 1, 0, 2, 0),
                 group2 = c(11, 11, 5, 8, 4))

## Change to long form
df_long <- df %>%
  tidyr::gather(key = "group",
                value = "value",
                group1, group2)

## Check Means and SD’s
df_long %>%
  dplyr::group_by(group) %>%
  furniture::table1(value)
df_long %>%
  ggplot(aes(x = group, y = value)) +
  geom_boxplot()
```

```
     group    value
[1]  group1   11
[2]  group1   1
[3]  group1   0
[4]  group1   2
[5]  group1   0
[6]  group2   11
[7]  group2   11
...  ...      ...
```

Example 1 - Output

	group	
	group1 n = 5	group2 n = 5
value		
	2.8 (4.7)	7.8 (3.3)



Example 1 – T-Test


```
df_long %>%
  car::leveneTest(value ~ group, data = ., center = "mean")

#> Levene's Test for Homogeneity of Variance (center = mean)
#>       Df F value Pr(>F)
#> group  1     .20   .667
#>        8

df_long %>%
  t.test(value ~ group,
         data = .,
         var.equal = TRUE)

#>      Welch Two Sample t-test
#> data:  value by group
#> t = -1.9642, df = 7.1732, p-value = 0.08927
#> alternative hypothesis: true difference in means is not equal to 0
#> 95 percent confidence interval:
#>   -10.9900148   0.9900148
#> sample estimates:
#> mean in group group1  mean in group group2
#>                  2.8                   7.8
```

Example 1


```
#> Levene's Test for Homogeneity of Variance (center = median)
#>       Df F value Pr(>F)
#> group  1       0      1
#>        8

#>         Welch Two Sample t-test
#> data:  value by group
#> t = -1.9642, df = 8, p-value = 0.08511
#> alternative hypothesis: true difference in means is not equal to 0
#> 95 percent confidence interval:
#>   -10.8701282   0.8701282
#> sample estimates:
#> mean in group group1  mean in group group2
#>                  2.8                  7.8
```

- After 6 months, the five participants taking the drug scored **numerically lower** on the depression scale (M = 2.80, SD = 4.66), compared their five counter parts taking placebo (M = 7.80, SD = 3.27).
- To test the effectiveness of the drug at reducing depression, an **independent samples *t*-test** was performed.
- The distribution of depression scores were **sufficiently normal** for the purposes of conducting a t-test (i.e. skew < |2.0| and kurtosis < |9.0|; Schmider, Sigler, Danay, Beyer, & Buhner, 2010).
- Additionally, the assumption of **homogeneity of variances** was tested and satisfied via Levene's *F*-test, *F*(4, 4) = .20, p = .667.
- The **independent samples *t*-test** did **not** find a statistically significant effect, *t*(8) = -1.964, p = .085.
- Thus, there is **no evidence** this drug reduces depression.

Assumptions (similar to 1-sample t-tests)

1. BOTH Samples were drawn **INDEPENDENTLY** at **random** (at least as representative as possible)
   - Nothing can be done to fix NON-representative samples!
   - Can not statistically test…violation: paired-samples t-test
2. The variable has a **NORMAL** distribution, for BOTH population
   - Not as important if the sample is large (Central Limit Theorem)
   - IF the sample is far from normal &/or small n, might want to transform variables
   - **Look at plots:** histogram, boxplot, & QQ plot (straight 45° line) ← sensitive to outliers!!!
     - **Skewness & Kurtosis:** Divided value by its *SE* & > ± 2 indicates issues
     - **Shapiro-Wilks test** (small N): $p < .05$ → not normal
     - **Kolmogorov-Smirnov test** (large N):
3. HOV = **Homogeneity of Variance**: BOTH populations have the sample spread
   - use Levene's F-test (null= HOV)

Random Assignment

- Random assignment to groups ↓ experimenter biases
  - Cases are enumerated
  - Numbers drawn and assigned to group in any of several ways
- Does not ensure equality of group characteristics
- Experiment: Random assignment & manipulation of IV
  - Treatment vs. control or 2 treatment groups
- Quasi-experiment: Either randomization or manipulation
- Non-experiment: Neither randomization or manipulation
  - Participants self-select or form naturally occurring groups

Violations of assumptions

- **Equal groups: Violations ‘hurt’ less**
  - Heterogeneity of variance
    - Small effects, p-value inaccurate ± .02
  - Non-normality
    - Small effects, p-value inaccurate ± .02
    - However: If samples are highly skewed or are skewed in opposite directions p-values can be *very* inaccurate
  - Both
    - Moderate effects if N is large, p-value can be inaccurate
    - Large effects if N is small, p-value can be *very* inaccurate

Violations of assumptions

- **Unequal groups: Violations 'hurt' more**
  - Heterogeneity of variance
    - Large effects
  - Non-normality
    - Large effects
  - Both
    - Huge effects

  - p-values can be **very** inaccurate with unequal ns and violations of assumptions, especially when N is small

Alternatives (assumptions violated)

- Violation of normality or ordinal DV
  - Two Sample Wilcoxon test (aka, Mann-Whitney *U* Test)
- Sample re-use methods
  - Rely on empirical, rather than theoretical, probability distributions
    - Exact statistical methods
    - Permutation and randomization tests
    - Bootstrapping

Confidence Intervals

- 95% *CI* for difference between means: $\mu_1 - \mu_2$
- Rearrange independent-samples *t*-test formula

$$CI_{1-\alpha} = \left(\bar{X}_1 - \bar{X}_2\right) \pm t_{\alpha/2} * (s_{\bar{X}_1 - \bar{X}_2})$$

- Estimation as NHST (Null hypothesis Significance Test)
  - If H_0: $\mu_1 = \mu_2$ and if *CI* does NOT contain 0, reject H_0
  - If H_0: $\mu_1 = \mu_2$ and if *CI* does contain 0, fail to reject H_0
- Compute for in-class example

Example 2

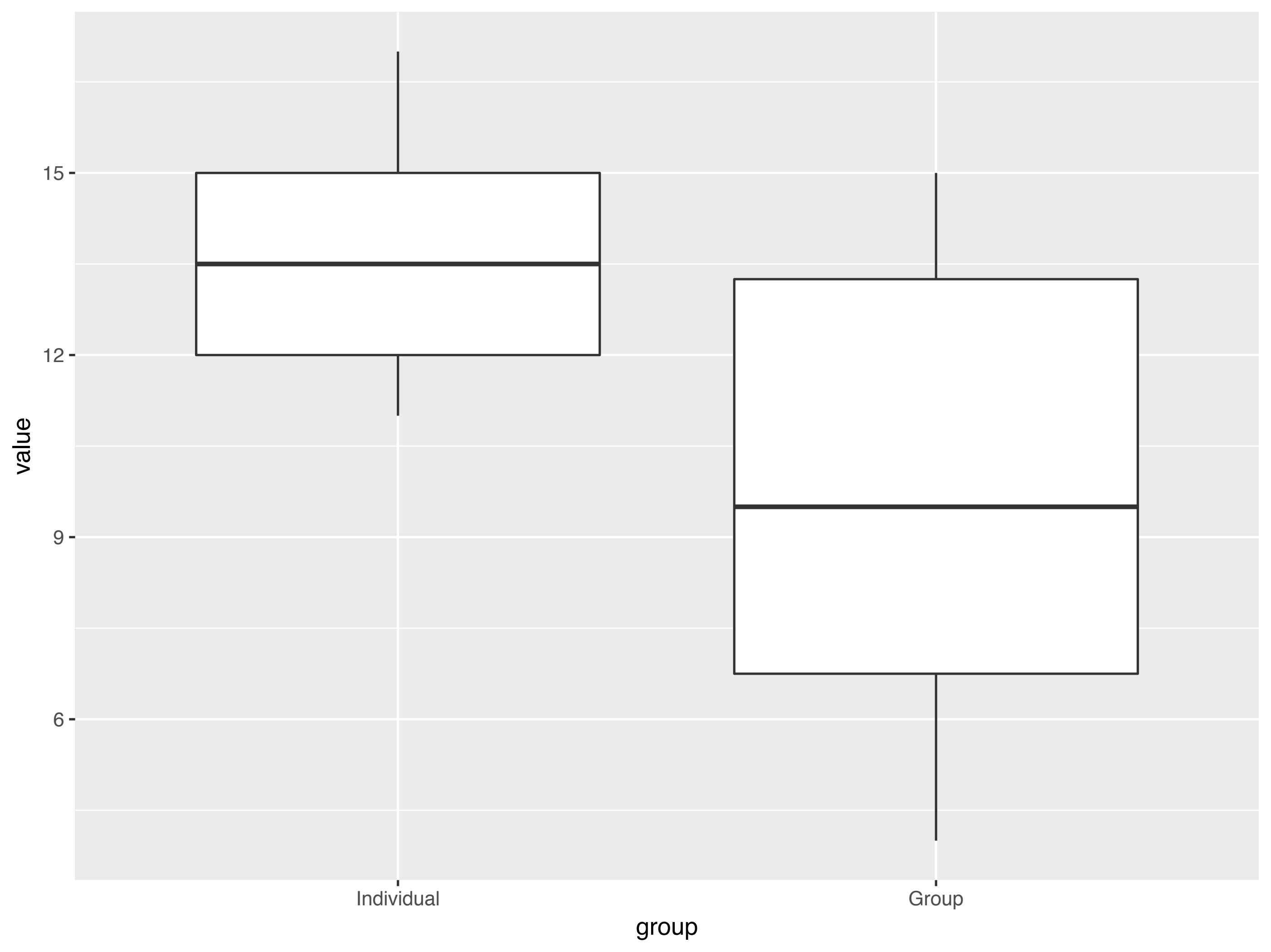
- An industrial psychologist is investigation the effects of different types of motivation on the performance of simulated clerical tasks. The 10 participants in the "individual motivation" sample are told that they will be rewarded according to how many tasks they successfully complete. The 12 participants in the "group motivation" sample are told that they will be rewarded according to the average number of tasks completed by all the participants in their sample. The number of tasks completed by each participant are as follows:
  - Individual Motivation: 11, 17, 14, 12, 11, 15, 13, 12, 15, 16
  - Group Motivation: 10, 15, 4, 8, 9, 14, 6, 15, 7, 11, 13, 5

```
## data object is df2_long
df2_long %>%
  dplyr::group_by(group) %>%
  furniture::table1(value)
```

	group	
	Individual	Group
	n = 10	n = 12
value		
	13.6 (2.1)	9.8 (3.9)

Example 2


```
library(tidyverse)
library(furniture)

## our data object is df2_long
## Check boxplots
df2_long %>%
  ggplot(aes(x = group, y = value)) +
  geom_boxplot()
```



Example 2


```
df2_long %>%
  car::leveneTest(value ~ group,
                  data = .,
                  center = "mean")

#> Levene's Test for Homogeneity of Variance (center = "mean")
#>        Df F value  Pr(>F)
#>  group  1  4.8287 0.03994 *
#>        20

df2_long %>%
  t.test(value ~ group,
         data = .,
         var.equal = FALSE)

#>       Welch Two Sample t-test
#> data:  value by group
#> t = 2.9456, df = 17.518, p-value = 0.008833
#> alternative hypothesis: true difference in means is not equal to 0
#> 95 percent confidence interval:
#>   1.098587 6.601413
#> sample estimates:
#> mean in group 1 mean in group 2
#>           13.60            9.75
```

Example 2


```
#> Levene's Test for Homogeneity of Variance (center = "mean")
#>        Df F value  Pr(>F)
#>  group  1  4.8287 0.03994 *
#>        20

#>          Welch Two Sample t-test
#> data:  value by group
#> t = 2.9456, df = 17.518, p-value = 0.008833
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#>   1.098587 6.601413sample estimates:
#> mean in group 1 mean in group 2
#>           13.60            9.75
```

- The number of tasks completed **was numerically higher** among the individually motivated group (n = 10, M = 13.60, SD = 2.12), compared to the individuals being motivated by the groups results (n = 12, M = 9.75, SD = 3.89).
- To test the difference in mean productivity, an **independent samples *t*-test** was performed.
- The distribution of depression scores were **sufficiently normal** for the purposes of conducting a t-test (i.e. skew < |2.0| and kertosis < |9.0|; Schmider, Sigler, Danay, Beyer, & Buhner, 2010).
- Additionally, the assumption of **homogeneity of variances** was tested and rejected via Levene's *F*-test, *F*(9, 11) = 4.829, p = .040.
- The independent samples, **separate variances *t*-test** found a statistically significant effect, *t*(17.52) = 2.946, p = .009.
- Thus, individual motivation **does** result in a mean 3.85 **additional tasks** completed compared to group motivation (95% CI: [1.01, 6.60]).