

TWO-WAY ANOVA

COHEN - CHAPTER 13 TEXTBOOK EXAMPLE

3 X 4 TWO-WAY ANOVA, COMPLEX MOTOR TASKS

DV (dependent variable)

- score on a video game that simulates driving a large truck at night

IV (categorical independent variables)

- **Sleep deprivation** (subjects spend four days in a sleep lab)
 - (1) **Control**: allowed to follow their own sleep schedule
 - (2) **Jet Lag**: keep usual amount, but not allow any during 11pm-7am
 - (3) **Interrupted**: usual amount, but no more than 2 hours at a time, separated by 1+ hours awake
 - (4) **Total Deprivation**: total lack of any sleep
- **Stimulant**
 - (1) **placebo**: sugar pill, but told it is caffeine
 - (2) **caffeine**: caffeine pill, told it is caffeine
 - (3) **reward**: mild electric shocks for mistakes & money for good performance
- **Subjects**
 - 5 per sleep x stimulant combination... $5 \times (4 \times 3) = 5 \times 12 = 60$

DATA

	Placebo	Caffeine	Reward
Control	24	26	28
	20	22	23
	29	20	24
	20	30	30
	28	27	33
Jet Lag	22	25	26
	18	31	20
	16	24	32
	25	27	23
Interrupted	27	21	30
	16	23	16
	20	28	13
	11	26	12
Total Lack	19	17	18
	14	19	19
	14	23	15
	17	16	11
	12	26	19
	18	18	11
	10	24	17

```
```{r}
data_wide <- data.frame(control_placebo = c(24, 20, 29, 20, 28),
 control_caffeine = c(26, 22, 20, 30, 27),
 control_reward = c(28, 23, 24, 30, 33),
 jetlag_placebo = c(22, 18, 16, 25, 27),
 jetlag_caffeine = c(25, 31, 24, 27, 21),
 jetlag_reward = c(26, 20, 32, 23, 30),
 interrup_placebo = c(26, 20, 11, 19, 14),
 interrup_caffeine = c(23, 28, 26, 17, 19),
 interrup_reward = c(16, 13, 12, 18, 19),
 totlack_placebo = c(14, 17, 12, 18, 10),
 totlack_caffeine = c(23, 16, 26, 18, 24),
 totlack_reward = c(15, 11, 19, 11, 17))
...
```
```

```
data_long <- data_wide %>%
  tidyr::gather(key = variable,
    value = score) %>%
  tidyr::separate(col = variable,
    into = c("sleep", "stimulant")) %>%
  dplyr::mutate(sleep = factor(sleep,
    levels = c("control",
      "jetlag",
      "interrup",
      "totlack"),
    labels = c("Own Schedule",
      "Jet Lag",
      "Interrupted",
      "Total Lack"))) %>%
  dplyr::mutate(stimulant = factor(stimulant,
    levels = c("placebo",
      "caffeine",
      "reward"),
    labels = c("Sugar Pill",
      "Caffeine Pill",
      "Electric Shock"))) %>%
  dplyr::mutate(id = row_number()) %>%
  dplyr::select(id, sleep, stimulant, score)
```

```
{r}  
data_long
```

| sleep
<fctr> | stimulant
<fctr> | score
<dbl> |
|-----------------|---------------------|----------------|
| Own Schedule | Sugar Pill | 24 |
| Own Schedule | Sugar Pill | 20 |
| Own Schedule | Sugar Pill | 29 |
| Own Schedule | Sugar Pill | 20 |
| Own Schedule | Sugar Pill | 28 |
| Own Schedule | Caffeine Pill | 26 |
| Own Schedule | Caffeine Pill | 22 |
| Own Schedule | Caffeine Pill | 20 |
| Own Schedule | Caffeine Pill | 30 |
| Own Schedule | Caffeine Pill | 27 |

1-10 of 60 rows

```
{r}  
glimpse(data_long)
```

```
Observations: 60  
Variables: 3  
$ sleep <fct> own schedule, own schedule, own schedule, own schedu  
$ stimulant <fct> Sugar Pill, sugar Pill, sugar Pill, sugar Pill, sugar  
$ score <dbl> 24, 20, 29, 20, 28, 26, 22, 20, 30, 27, 28, 23, 24, 3
```

| | sleep
↑ | stimulant
↕ | score
↕ |
|----|--------------|----------------|------------|
| 1 | Own Schedule | Sugar Pill | 24 |
| 2 | Own Schedule | Sugar Pill | 20 |
| 3 | Own Schedule | Sugar Pill | 29 |
| 4 | Own Schedule | Sugar Pill | 20 |
| 5 | Own Schedule | Sugar Pill | 28 |
| 6 | Own Schedule | Caffeine Pill | 26 |
| 7 | Own Schedule | Caffeine Pill | 22 |
| 8 | Own Schedule | Caffeine Pill | 20 |
| 9 | Own Schedule | Caffeine Pill | 30 |
| 10 | Own Schedule | Caffeine Pill | 27 |
| 11 | Own Schedule | Electric Shock | 28 |
| 12 | Own Schedule | Electric Shock | 23 |
| 13 | Own Schedule | Electric Shock | 24 |
| 14 | Own Schedule | Electric Shock | 30 |
| 15 | Own Schedule | Electric Shock | 33 |
| 16 | Jet Lag | Sugar Pill | 22 |
| 17 | Jet Lag | Sugar Pill | 18 |
| 18 | Jet Lag | Sugar Pill | 16 |
| 19 | Jet Lag | Sugar Pill | 25 |
| 20 | Jet Lag | Sugar Pill | 27 |

MARGINAL MEAN'S

| | Placebo | Caffeine | Reward | |
|-------------|---------|----------|--------|--------|
| Control | 24.2 | 25.0 | 27.6 | 25.6 |
| Jet Lag | 21.6 | 25.6 | 26.2 | 24.47 |
| Interrupted | 16.0 | 22.6 | 15.6 | 18.07 |
| Total Lack | 14.2 | 21.4 | 14.6 | 16.73 |
| | 19.0 | 23.65 | 21.0 | 21.217 |

```
data_long %>%
  dplyr::group_by(sleep) %>%
  furniture::table1(score)
```

| | sleep | | | |
|-------|------------------------|-------------------|-----------------------|----------------------|
| | Own Schedule
n = 15 | Jet Lag
n = 15 | Interrupted
n = 15 | Total Lack
n = 15 |
| score | 25.60 (4.12) | 24.47 (4.63) | 18.07 (4.86) | 16.73 (4.86) |

```
data_long %>%
  dplyr::group_by(stimulant) %>%
  furniture::table1(score)
```

| | stimulant | | |
|-------|----------------------|-------------------------|--------------------------|
| | Sugar Pill
n = 20 | Caffeine Pill
n = 20 | Electric Shock
n = 20 |
| score | 19.00 (5.55) | 23.65 (4.20) | 21.00 (7.10) |

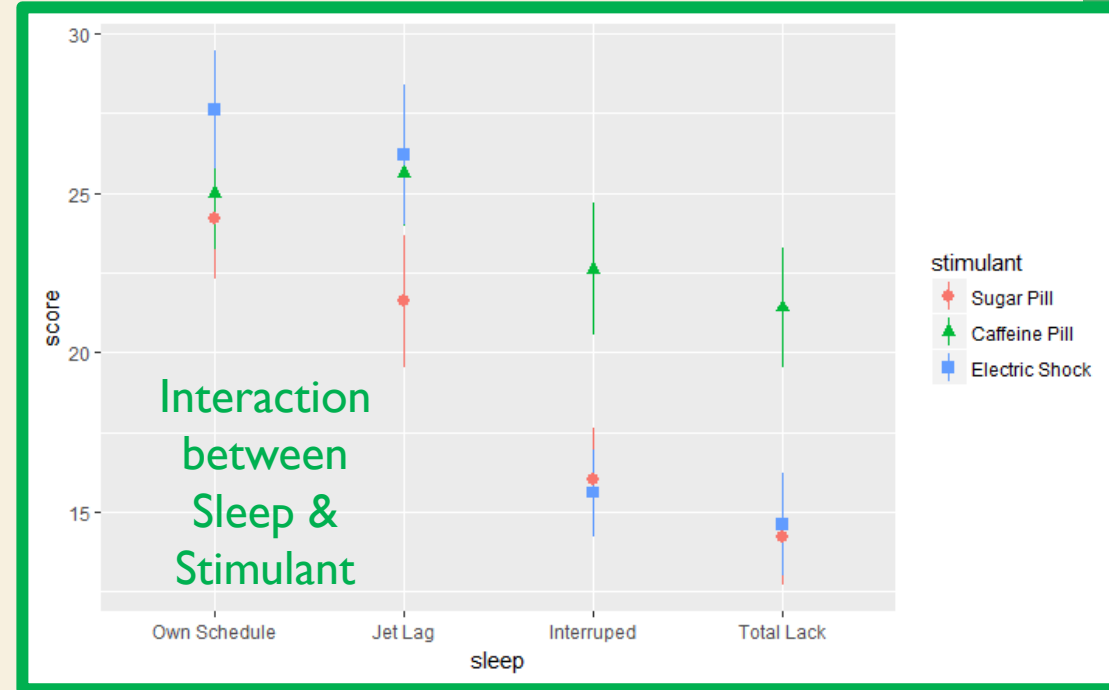
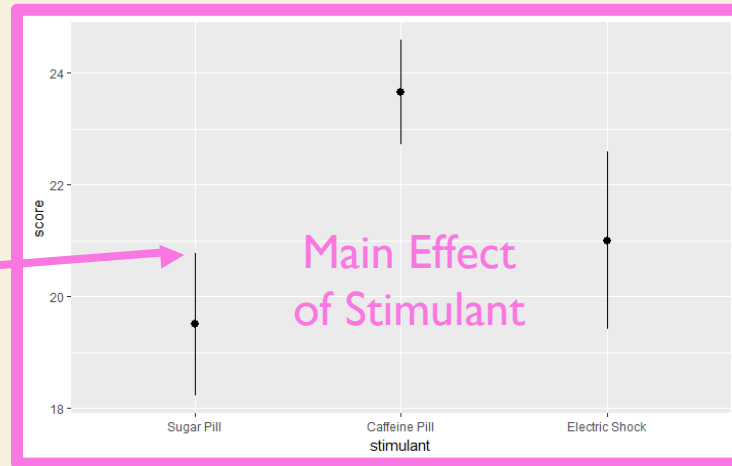
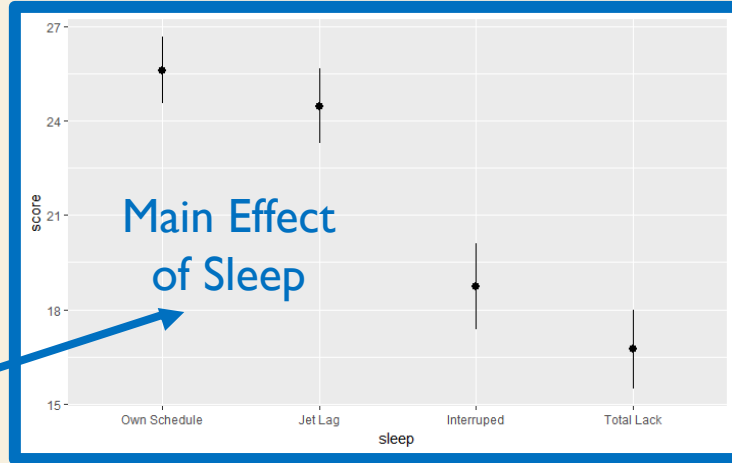
```
data_long %>%
  dplyr::group_by(sleep, stimulant) %>%
  dplyr::summarise(mean = mean(score)) %>%
  tidyr::spread(key = stimulant,
                 value = mean)
```

| sleep
<fctr> | Sugar Pill
<dbl> | Caffeine Pill
<dbl> | Electric Shock
<dbl> |
|-----------------|---------------------|------------------------|-------------------------|
| Own Schedule | 24.2 | 25.0 | 27.6 |
| Jet Lag | 21.6 | 25.6 | 26.2 |
| Interrupted | 16.0 | 22.6 | 15.6 |
| Total Lack | 14.2 | 21.4 | 14.6 |

PLOT OF MEAN'S

```
data_long %>%
  ggplot(aes(x = sleep,
             y = score)) +
  stat_summary()
```

| | Placebo | Caffeine | Reward | |
|------|---------|----------|--------|--------|
| Norm | 24.2 | 25.0 | 27.6 | 25.6 |
| Lag | 21.6 | 25.6 | 26.2 | 24.47 |
| Int | 16.0 | 22.6 | 15.6 | 18.07 |
| None | 14.2 | 21.4 | 14.6 | 16.73 |
| | 19.0 | 23.65 | 21.0 | 21.217 |

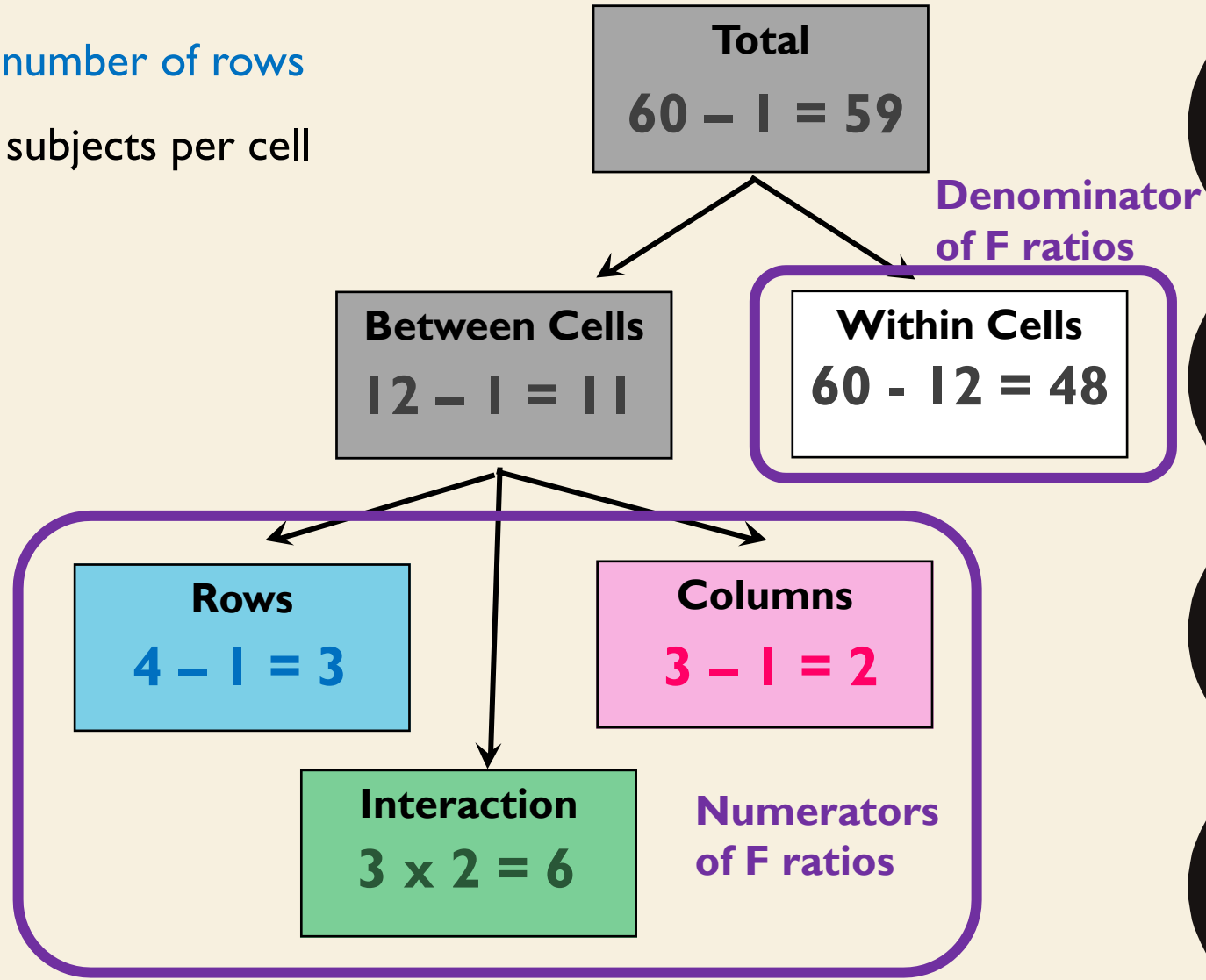


```
data_long %>%
  ggplot(aes(x = stimulant,
             y = score)) +
  stat_summary()
```

c = number of columns

r = number of rows

n = subjects per cell



DEGREES OF FREEDOM

2-WAY ANOVA

```
aov_2way <- data_long %>%  
  afex::aov_4(score ~ sleep*stimulant + (1|id),  
             data = .)
```

| | SS | Df | MS | F | p |
|-------------------------|---------|----|--------------|---------------|-----------------|
| Between-Cells | 1309.38 | 11 | | | |
| SLEEP Row Groups | 896.98 | 3 | 298.99 | 18.241 | <.001 |
| STIM Column Groups | 217.63 | 2 | 108.82 | 6.639 | .003 |
| INTER (Row x Col) | 197.77 | 6 | 32.46 | 1.980 | .087 |
| Within-Cells (Residual) | 786.80 | 48 | 16.39 | | |
| Total | 2096.18 | 59 | | | |

IF THE
INTERACTION IS
SIGNIFICANT...

MAIN EFFECTS
SHOULD NOT BE
INTERPRETED IN
ISOLATION

```
{r}  
aov_2way$Anova
```

Anova Table (Type III tests)

Response: dv

| | Sum Sq | Df | F value | Pr(>F) | |
|-----------------|---------|----|-----------|-----------|-----|
| (Intercept) | 27008.8 | 1 | 1647.7163 | < 2.2e-16 | *** |
| sleep | 897.0 | 3 | 18.2406 | 4.896e-08 | *** |
| stimulant | 217.6 | 2 | 6.6385 | 0.002849 | ** |
| sleep:stimulant | 194.8 | 6 | 1.9803 | 0.087003 | . |
| Residuals | 786.8 | 48 | | | |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

INTERACTION CONTRASTS

$$2 \times 2: df_{con} = (r - 1)(c - 1) = 1$$

If the interaction is significant...
pick TWO PAIRS to compare (extreme)

Does **CAFFEINE** have the same effect in **NO SLEEP** as **NORMAL SLEEP**?

$$c_i = 1, -1, -1, 1$$

$$\sum c_i^2 = 4$$

Caffeine's Effect in Normal

Caffeine's Effect in None

| | Placebo | Caffeine | Reward | |
|------|---------|----------|--------|--------|
| Norm | 24.2 | 25.0 | 27.6 | 25.6 |
| Lag | 21.6 | 25.6 | 26.2 | 24.47 |
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$$L = (\bar{X}_{11} - \bar{X}_{12}) - (\bar{X}_{21} - \bar{X}_{22})$$

$$L = \bar{X}_{11} - \bar{X}_{12} - \bar{X}_{21} + \bar{X}_{22}$$

$$L = 1 \cdot \bar{X}_{11} + -1 \cdot \bar{X}_{12} + -1 \cdot \bar{X}_{21} + 1 \cdot \bar{X}_{22}$$

$$L = 1 \cdot \quad + -1 \cdot \quad + -1 \cdot \quad + 1 \cdot$$

$$L = 24.2 - 25.0 - 14.2 + 21.4 = 6.4$$

24.2 25.0

14.2 21.4

$$SS_{con} = \frac{n \cdot L^2}{\sum c_i^2}$$

$$F_{con} = \frac{SS_{con}}{MS_W}$$

$$SS_{con} = \frac{5 \cdot 6.4^2}{1 + 1 + 1 + 1} = \frac{5 \cdot 40.96}{4} = \frac{204.8}{4} = 51.2$$

$$F_{Scheffe} = df_{int} \cdot F(df_{int}, df_W)$$

$$F_{con} = \frac{51.2}{16.39} = 3.12$$

$$F_{Scheffe} = 6 \cdot F(6, 48) = 6 \cdot F_{.05}(6, 40) = 6 \cdot 2.34 = 14.04$$

INTERACTION CONTRASTS

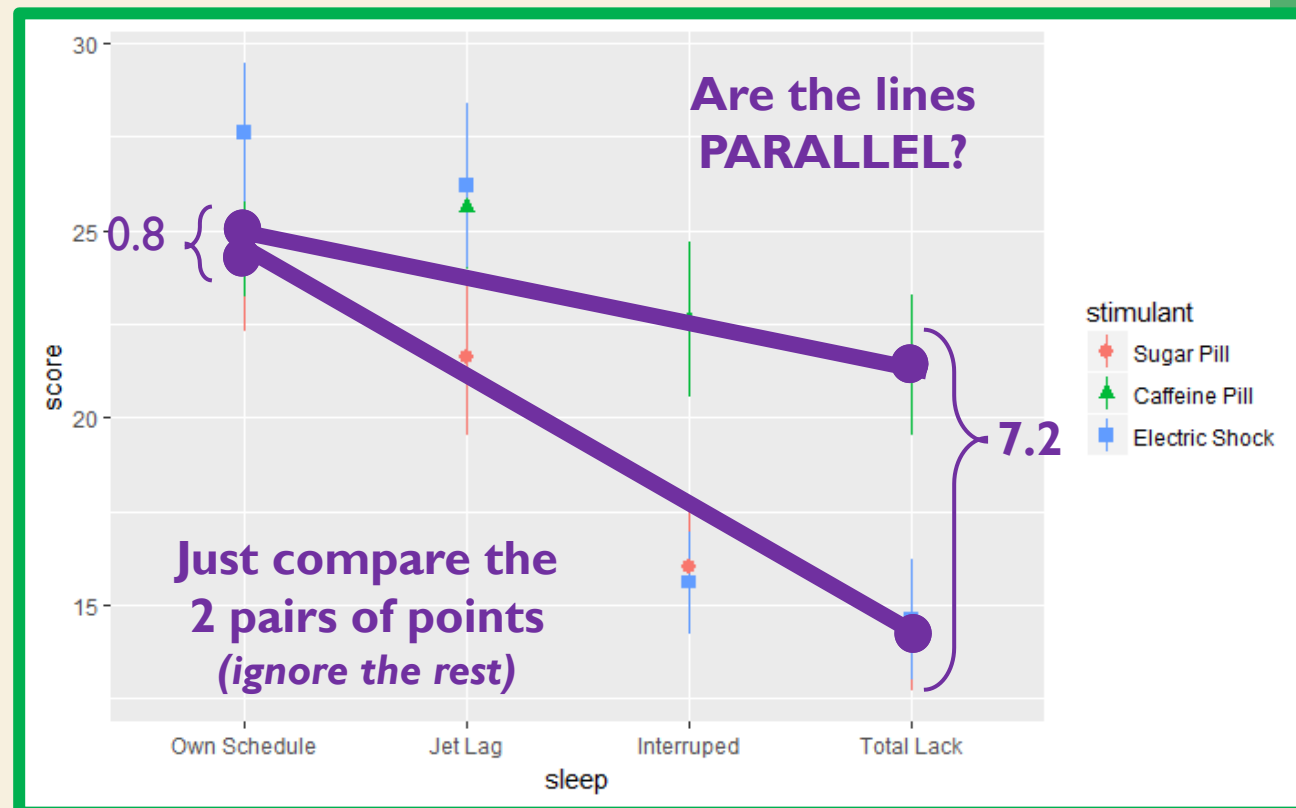
Does **CAFFEINE** have the same effect in **NO SLEEP** as **NORMAL SLEEP**?

| | Placebo | Caffeine | Reward | |
|------|---------|----------|--------|--------|
| Norm | 24.2 | 25.0 | 27.6 | 25.6 |
| Lag | 21.6 | 25.6 | 26.2 | 24.47 |
| Int | 16.0 | 22.6 | 15.6 | 18.07 |
| None | 14.2 | 21.4 | 14.6 | 16.73 |
| | 19.0 | 23.65 | 21.0 | 21.217 |

$$F_{contrast} = 3.12$$

Vs

$$F_{Scheffe} = 14.04$$



This provides no evidence

that **CAFFEINE** has a different effect

in **NORMAL SLEEP** vs **TOTAL DEPRIVATION**.