EDUC/PSY 6600 Formula Sheet

Comparing Means

X = measurement in sample

μ = POPULATION MEAN

Situation		Estimate	C.V.	$SE_{estimate}$	Hypothesis	Test-statistic (df)	Notes
		Confidence Intervals			Hypothesis Testing		
1 SAMPLE		$\bar{x}$ Sample mean	± z*	$SE_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$	$H_0: \mu_x = \mu_0$ $H_a: \mu_x [\neq><] \mu_0$	$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$	Use if you know the population SD or when sample is very large
			± t*	$SE_{\bar{x}} = \frac{s}{\sqrt{n}}$		$t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$ $df = n - 1$	Use with a small sample or using the sample's SD instead of the population's $d = \bar{X} / SE_{\bar{x}}$
MATCHED PAIRS	$(n = \# \ of \ pairs)$	$D = x_1 - x_2$	± t*	$SE_{\bar{D}} = \frac{s_D}{\sqrt{n}}$	$H_0: \mu_D = \mu_0$ $H_a: \mu_D [\neq><] \mu_0$ ---or— $H_0: \mu_1 - \mu_2 = 0$ $H_a: \mu_1 - \mu_2 [\neq><]\ 0$	$t = \frac{\bar{D} - \mu_0}{s_D / \sqrt{n}}$ $df = n - 1$	**"Direct-Differences Method"** First must subtract all pairs to create a new variable D and then find s_D which is the SD of D's
		$\bar{D} = \overline{x_1} - \overline{x_2}$	± t*	$SE_{\bar{D}} = \sqrt{\frac{s_1^2 + s_2^2}{n} - \frac{2 r s_1 s_2}{n}}$		$t = \frac{\bar{D} - \mu_0}{\sqrt{\frac{s_1^2 + s_2^2}{n} - \frac{2 r s_1 s_2}{n}}}$ $df = n - 1$	**"Correlation Method"** Instead of subtracting all pairs, find each variables' M & SD, as well as the r between the variables in the sample
2 INDEPENDENT SAMPLES	(not paired)	$\bar{D} = \overline{x_1} - \overline{x_2}$ Difference in 2 sample means	± z*	$SE_{\bar{D}} = \sqrt{\frac{{\sigma_1}^2}{n_1} + \frac{{\sigma_2}^2}{n_2}}$	$H_0: \mu_1 - \mu_2 = 0$ $H_a: \mu_1 - \mu_2 [\neq><]\ 0$	$z = \frac{\bar{D}}{\sqrt{\frac{{\sigma_1}^2}{n_1} + \frac{{\sigma_2}^2}{n_2}}}$	Know both populations SD or when samples are large
			± t*	$SE_{\bar{D}} = \sqrt{\frac{{s_1}^2}{n_1} + \frac{{s_2}^2}{n_2}}$		$t = \frac{\bar{D}}{\sqrt{\frac{{s_1}^2}{n_1} + \frac{{s_2}^2}{n_2}}}$ $\min(n's) - 1 < df_{SV} < n_1 + n_2 - 2$	**"Separate Variances t-test"** Use with equal n's –or- if violated HOV (var.equal = FALSE). Also use with equal n's if the larger sample as the smaller SD $d = \bar{D} / SE_{\bar{D}}$
				$SE_{\bar{D}} = \sqrt{s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$	$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$ $s_p^2 \xRightarrow{n_1 = n_2,} \frac{s_1^2 + s_2^2}{2}$	$t = \frac{\bar{D}}{\sqrt{s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$ $df_{pooled} = n_1 + n_2 - 2$	**"Pooled Variance t-test"** Assumes the populations have equal SD's (test HOV w/Levene's Test, var.equal = TRUE). Also use then the n's are not equal and the larger sample has the larger SD $d = \bar{D} / s_p$

Effect Size

$$\delta = \text{"expected t or z"} \ \left(\textbf{population}\ parameters\right)$$

$$1\ group\!: \ \delta = \frac{\mu}{\sigma}\sqrt{n} \xrightarrow{d=\frac{\mu}{\sigma}} \delta = d\sqrt{n}$$

$$2\ groups\!: \ \delta \overset{n_1=n_2,}{\Longrightarrow} \frac{\mu_1-\mu_2}{\sigma}\sqrt{\frac{n}{2}} \xrightarrow{d=\frac{\mu_1-\mu_2}{\sigma}} \delta = d\sqrt{\frac{n}{2}}$$

$$\overset{est.\ \mathbf{d}=g\left(1-\frac{3}{4\,df-1}\right)}{\Longleftrightarrow}$$

$$g = \text{"}effect\ size\text{"} \ \left(\textbf{sample}\ statistics\right)$$

$$g = \frac{\bar{X}_1-\bar{X}_2}{s_p} = \begin{cases} \overset{n_1=n_2,}{\Longrightarrow} t\sqrt{\frac{2}{n}} \\ \overset{n_1\neq n_2,}{\Longrightarrow} t\sqrt{\frac{n_1+n_2}{n_1 n_2}} \end{cases}$$

$$Pearson's\ Correlation\!: \ r = \frac{\sum_{i=1}^{N} z_x z_y}{N}$$

Post Hoc Test (after ANOVA)

Pairwise	Linear Contrast
$t_{pair} = \dfrac{\bar{x}_i-\bar{x}_j}{\sqrt{\dfrac{2MS_w}{n}}}$	$L = \sum c_i\bar{x}_i \xrightarrow{0=\sum c_i} SS_{con} = \dfrac{nL^2}{\sum {c_i}^2} \rightarrow F_{con(1,df_w)} = \dfrac{SS_{con}}{MS_w}$
Fisher's LSD $LSD = t_{CV}\sqrt{\dfrac{2MS_w}{n}}$	Scheffe's F, 1-way $F_s = (k-1)F_{cv}(k-1, n_T-k)$
Tukey's HSD $HSD = q_{CV}\sqrt{\dfrac{MS_w}{n}}$	Scheffe's F, 2-way $F_s = df_{int}\,F_{cv}(df_{int}, df_w)$

($\bar{x}_G = grand\ mean$, $n_T = total\ sample\ size$, $n = cell\ size$, $n_r = \#in\ row$, $n_c = \#in\ column$)

One-Way ANOVA

Source	SS	df	MS	F	p
Between-Groups ($k = \#\ groups$)	$df_{BetGrp}MS_{BetGrp}$	$k-1$	$n\dfrac{\sum_{i=1}^{k}(\bar{x}_i-\bar{x}_G)^2}{k-1}$	$\dfrac{MS_{BetGrp}}{MS_{WithGrp}}$	
Within-Groups (**Residual or Error**)	$df_{WithGrp}MS_{WithGrp}$	n_T-k	$\boxed{\dfrac{\sum_{i=1}^{k}s_i^2}{k}}$		
Total	$SS_{BetGrp}+SS_{WithGrp}$	n_T-1			

$$ordinary\ \eta^2 = \frac{SS_{BetGrp}}{SS_{total}}$$

$$modified\ \eta^2 = \eta^2\left(1-\frac{1}{F}\right)$$

$$est.\varpi^2 = \frac{SS_{BetGrp} - (k-1)MS_w}{SS_{total}+MS_w}$$

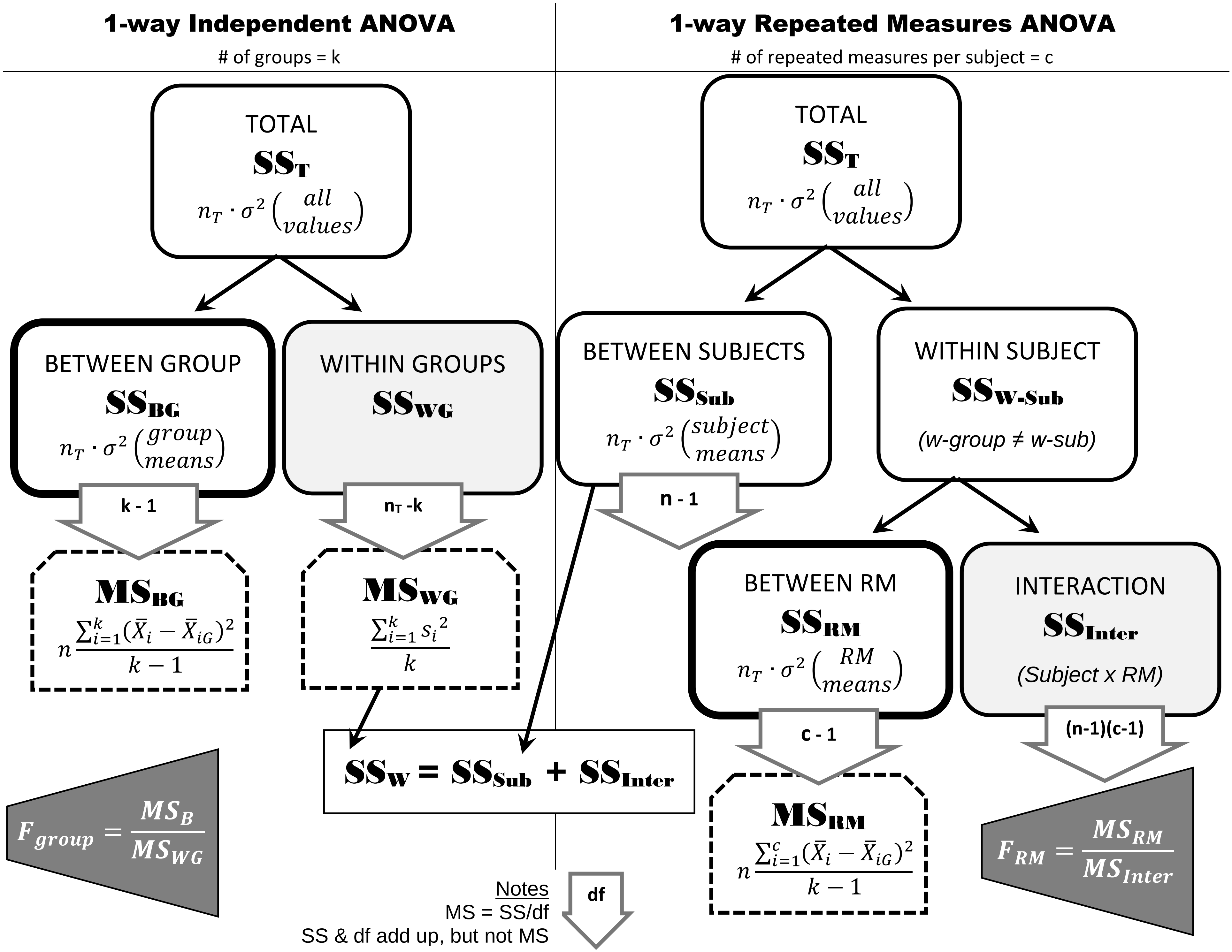
Two-Way ANOVA

Source	SS	df	MS	F	p
Between-Cells (**Row, Cell, & Interaction**)	$n_T\dfrac{\sum_{i=1}^{r}\sum_{j=1}^{c}\left(\bar{x}_{ij}-\bar{x}_G\right)^2}{rc}$	$rc-1$	$SS_{total} = SS_{BetRow}+SS_{BetCol}+SS_{Inter}+SS_{error}$		
Row Groups ($r = \#\ rows$)	$n_T\dfrac{\sum_{i=1}^{r}(\bar{x}_i-\bar{x}_G)^2}{r}$	$r-1$	$n_r\dfrac{\sum_{i=1}^{r}(\bar{x}_i-\bar{x}_G)^2}{r-1}$	$\dfrac{MS_{BetRow}}{MS_{WithCell}}$	
Column Groups ($c = \#\ columns$)	$n_T\dfrac{\sum_{j=1}^{c}\left(\bar{x}_j-\bar{x}_G\right)^2}{c}$	$c-1$	$n_c\dfrac{\sum_{j=1}^{c}\left(\bar{x}_j-\bar{x}_G\right)^2}{c-1}$	$\dfrac{MS_{BetCol}}{MS_{WithCell}}$	
INTER (Row x Col)	$SS_{BetCells}$ $-SS_{BetRow}$ $-SS_{BetCol}$	$(r-1)(c-1)$	$\dfrac{SS_{Inter}}{df_{inter}}$	$\dfrac{MS_{Inter}}{MS_{WithCell}}$	
Within-Cells (**Residual or Error**)	$df_{WithCell}MS_{WithCell}$	n_T-rc	$\boxed{\dfrac{\sum_{i=1}^{r}\sum_{j=1}^{c}s_{ij}^2}{rc}}$		
Total	$SS_{BetCell}+SS_{WithCell}$	n_T-1			

$$ordinary\ \eta^2 = \frac{SS_{Effect}}{SS_{total}}$$

$$partial\ \eta^2 = \frac{SS_{Effect}}{SS_{Effect}+SS_{error}}$$

$$est.\varpi^2 = \frac{SS_{Effect} - \left(df_{Effect}\right)MS_w}{SS_{total}+MS_w}$$


1-way Independent ANOVA
# of groups = k
1-way Repeated Measures ANOVA
# of repeated measures per subject = c
TOTAL
SS_T
$n_T \cdot \sigma^2 \binom{all}{values}$
BETWEEN GROUP
SS_{BG}
$n_T \cdot \sigma^2 \binom{group}{means}$
WITHIN GROUPS
SS_{WG}
k - 1
n_T -k
MS_{BG}
$n \frac{\sum_{i=1}^{k} (\bar{X}_i - \bar{X}_{iG})^2}{k-1}$
MS_{WG}
$\frac{\sum_{i=1}^{k} s_i{}^2}{k}$
$F_{group} = \frac{MS_B}{MS_{WG}}$
TOTAL
SS_T
$n_T \cdot \sigma^2 \binom{all}{values}$
BETWEEN SUBJECTS
SS_{Sub}
$n_T \cdot \sigma^2 \binom{subject}{means}$
WITHIN SUBJECT
$SS_{W\text{-}Sub}$
(w-group ≠ w-sub)
n - 1
BETWEEN RM
SS_{RM}
$n_T \cdot \sigma^2 \binom{RM}{means}$
INTERACTION
SS_{Inter}
(Subject x RM)
c - 1
(n-1)(c-1)
$SS_W = SS_{Sub} + SS_{Inter}$
MS_{RM}
$n \frac{\sum_{i=1}^{c} (\bar{X}_i - \bar{X}_{iG})^2}{k-1}$
$F_{RM} = \frac{MS_{RM}}{MS_{Inter}}$
Notes
MS = SS/df
SS & df add up, but not MS
df

ANOVA: df trees

1-way or independent groups (Ch 12)

n = # obs per group

k = # groups

n_T = total # observations

$$\eta^2 = \frac{SS_{BetGrp}}{SS_{total}}$$

- **df_{TOTAL}** $(n_T - 1)$
  - **$df_{Bet\text{-}group}$** $(k - 1)$
  - **$df_{With\text{-}group}$** $(n_T - k)$

2-way or factorial (Ch 14)

n = # obs per cell

r = # groups - rows

c = # groups - columns

n_T = total # observations

$$\eta^2_{ord} = \frac{SS_{Effect}}{SS_{total}}$$

$$\eta^2_p = \frac{SS_{Effect}}{SS_{Effect} + SS_{WithCell}}$$

- **df_{TOTAL}** $(n_T - 1)$
  - **$df_{Bet\text{-}Cell}$** $(rc - 1)$
    - **df_{Row}** $(r - 1)$
    - **$df_{Row\ x\ Col}$** $(r - 1)*(c - 1)$
    - **df_{Col}** $(c - 1)$
  - **$df_{With\text{-}Cell}$** $(n_T - rc)$

1-way Repeated Measures (Ch 15)

n = # subjects

c = # levels of the RM (# time points)

n_T = total # observations

$$\eta^2_{RM} = \frac{SS_{RM}}{SS_{RM} + SS_{S*RM}}$$

- **df_{TOTAL}** $(n_T - 1)$
  - **$df_{Bet\text{-}Sub}$** $(n - 1)$
  - **$df_{With\text{-}Sub}$** $n(c - 1)$
    - **Df_{RM}** $(c - 1)$
    - **Df_{SubxRM}** $(n - 1)(c - 1)$

Mixed Design (Ch 16)

n = # subjects

k = # groups

c = # levels of the RM (# times)

n_T = total # observations

$$Group: \eta^2_{Gen} = \frac{SS_{Grp}}{SS_{Grp} + SS_{WithGrp} + SS_{S*RM}}$$

$$Grp\ assigned: \eta^2_p = \frac{SS_{Grp}}{SS_{Grp} + SS_{WithGrp}}$$

$$Grp\ observed: \eta^2_p = \frac{SS_{Grp}}{SS_{Total} - SS_{RM}}$$

$$Grp * RM: \eta^2_p = \frac{SS_{Grp*RM}}{SS_{Grp*RM} + SS_{S*RM}}$$

$$RM: \eta^2_p = \frac{SS_{RM}}{SS_{RM} + SS_{S*RM}}$$

- **df_{TOTAL}** $(n_T - 1)$
  - **$df_{Bet\text{-}Sub}$** $(nk - 1)$
    - **df_{Group}** $(k - 1)$
    - **$df_{With\text{-}group}$** $k(n - 1)$
  - **$df_{With\text{-}Sub}$** $nk(c - 1)$
    - **df_{RM}** $(c - 1)$
    - **$df_{Group\ x\ RM}$** $(k - 1)*(c - 1)$
    - **$df_{Sub\ x\ RM}$** $k(c - 1)(n - 1)$