

## Chapter 14

# FACTORIAL ANOVA

*‘People can be divided into two classes:  
Those who go ahead and do something,  
and those who sit still and inquire,  
‘Why wasn’t it done the other way?’*

– Oliver Wendell Holmes, American Physician, Writer,  
Humorist, Harvard Professor, 1809-1894



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*Dr. Petrov is interested in conducting an experiment where:*

- *30 high school students are randomly assigned to a new **computer simulation tool** for learning geometry and*
- *30 other students are randomly assigned to the standard **lecture and paper/pencil problem** solving format.*

*However, Dr. Petrov is also interested in the **effect of sex** differences on learning outcomes.*

# ANALYSIS OF VARIANCE

- ANOVA types...
  - 1-Way ANOVA = 1 factor
  - 2-Way ANOVA = 2 factors (focus of lecture)
  - 3-Way ANOVA = 3 factors
  - 4-Way ANOVA = 4 factors
- # levels of each factor determines ANOVA design
  - # Levels: Row factor = 2, Column factor = 3
    - 2-way ANOVA, 2X3 factorial design
  - # Levels: Row factor = 4, Column factor = 3
    - 2-way ANOVA, 4X3 factorial design



# FACTORIAL 2-WAY ANOVA

- **Simultaneously evaluate effect of 2 or more factors on continuous outcome**
  - Cross-classification
  - Participants only belong to 1 mutually exclusive 'cell'
    - Within 1 level of row factor and 1 level of columns factor

- Typical 2-way ANOVA
  - 3x2 design
  - Row factor (A): 3 levels
  - Column factor (B): 2 levels

		B	
		B1	B2
A	A1	11	12
	A2	21	22
	A3	31	32

# TEST OF ROW MAIN EFFECT

- Do **row marginal** means differ?
  - *Do population means differ across levels of row factor, averaging across levels of column factor?*
  - $H_0: \mu_{j1} = \mu_{j2} = \mu_{jr}$
  - $H_1: \text{Not } H_0$

		B		Marginals
		B1	B2	
A	A1	$M_{11}$	$M_{12}$	$M_{A1}$
	A2	$M_{21}$	$M_{22}$	$M_{A2}$
	A3	$M_{31}$	$M_{32}$	$M_{A3}$
Marginals		$M_{B1}$	$M_{B2}$	

# TEST OF ROW MAIN EFFECT

- Do **column marginal** means differ?
  - Do population means differ across levels of column factor, averaging across levels of row factor?
  - $H_0: \mu_{j1} = \mu_{j2} = \mu_{jr}$
  - $H_1: \text{Not } H_0$

		B		Marginals
		B1	B2	
A	A1	$M_{11}$	$M_{12}$	$M_{A1}$
	A2	$M_{21}$	$M_{22}$	$M_{A2}$
	A3	$M_{31}$	$M_{32}$	$M_{A3}$
Marginals		$M_{B1}$	$M_{B2}$	

# TEST OF INTERACTION EFFECT

- Does pattern of cell means differ?
  - Are differences among population means across row factor similar across all levels of column factor (and vice versa)?
  - $H_0$ : Differences among levels for 1 factor do not vary across levels of other factor
  - $H_1$ : Not  $H_0$

		B		Marginals
		B1	B2	
A	A1	$M_{11}$	$M_{12}$	$M_{A1}$
	A2	$M_{21}$	$M_{22}$	$M_{A2}$
	A3	$M_{31}$	$M_{32}$	$M_{A3}$
Marginals		$M_{B1}$	$M_{B2}$	

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A	A1	$M_{11}$	$M_{12}$	$M_{A1}$
	A2	$M_{21}$	$M_{22}$	$M_{A2}$
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Marginals		$M_{B1}$	$M_{B2}$	

# POSSIBLE OUTCOMES

1. No significant main effects or interaction(s)
2. No significant interaction
  - Significant main effect for rows, but not for columns
  - Significant main effect for columns, but not for rows
  - Significant main effects for both rows and columns
3. Significant interaction and...
  - Non-significant main effects for rows or columns
  - Significant main effect for rows, but not for columns
  - Significant main effect for columns, but not for rows
  - Significant main effects for both rows and columns



# REDUCED ERROR

Subject-to-subject variability contributes to increased  $MS_W$   
= Less power

Adding factors that explain subject-to-subject variability in outcome reduces  $MS_W$  and increases power

Variance within (and thus across) individual cells is reduced as cases become more homogeneous in terms of their characteristics

Factors that do not have this effect may slightly decrease power

$df_W$  (which =  $N - rc$ ) decreases as # cells increases, increasing  $MS_W$  and decreasing  $F$ -ratios

## Alternatives

Restriction (subjects from 1-level only – reduced generalizability)

Repeated-measures (matched) designs

# ASSUMPTIONS

## Similar to 1-Way ANOVA

- Independence
- Outcome is normally distributed in population
- Homogeneity of variance
  - Variances within each cell are equal

# VARIANCE COMPONENTS

- $SS_{Total}$  partitioned into 4 components

$$SS_{Total} = SS_{(R)ows} + SS_{(C)olumns} + SS_{RC} + SS_{Within}$$

- When balanced, previous  $SS_B$  from 1-Way ANOVA partitioned into 3 components:  $R, C, RC$
- 1-way ANOVA uses groups and factorial ANOVA uses cells to compute  $SS$
- Following equations are for balanced designs

# $SS_R$

- In computing row means all scores in a given row are averaged regardless of column
  - $n_{row}$  = # participants per row

$$SS_R = n_{row} [(\bar{X}_{row1} - \bar{X}_{GM})^2 + (\bar{X}_{row2} - \bar{X}_{GM})^2 + \dots + (\bar{X}_{row r} - \bar{X}_{GM})^2]$$

$$SS_R = \frac{\left(\sum_{i=1}^n X_{row1}\right)^2 + \left(\sum_{i=1}^n X_{row2}\right)^2 + \dots + \left(\sum_{i=1}^n X_{row r}\right)^2}{n_{row}} - \frac{\left(\sum_{i=1}^n X\right)^2}{N}$$

# SS<sub>C</sub>

- In computing column means all scores in a given column are averaged regardless of row
  - $n_{col}$  = # participants per column

$$SS_C = n_{column} [(\bar{X}_{col1} - \bar{X}_{GM})^2 + (\bar{X}_{col2} - \bar{X}_{GM})^2 + \dots + (\bar{X}_{col r} - \bar{X}_{GM})^2]$$

$$SS_C = \frac{\left(\sum_{i=1}^n X_{col1}\right)^2 + \left(\sum_{i=1}^n X_{col2}\right)^2 + \dots + \left(\sum_{i=1}^n X_{col r}\right)^2}{n_{col}} - \frac{\left(\sum_{i=1}^n X\right)^2}{N}$$



# $SS_{RC}$

- Variability among cell means when variability due to individual row and column effects have been removed

$$SS_{RC} = n_{cell} [(\bar{X}_{cell11} - \bar{X}_{GM})^2 + (\bar{X}_{cell12} - \bar{X}_{GM})^2 + \dots + (\bar{X}_{cellrc} - \bar{X}_{GM})^2] - SS_R - SS_C$$

$$SS_{RC} = \frac{\left(\sum_{i=1}^n X_{cell11}\right)^2 + \left(\sum_{i=1}^n X_{cell12}\right)^2 + \dots + \left(\sum_{i=1}^n X_{cellrc}\right)^2}{n_{cell}} - \frac{\left(\sum_{i=1}^n X\right)^2}{N} - SS_R - SS_C$$

# $SS_W$

- $SS$  within each cell added together
  - $SS_W = SS_{I1} + SS_{I2} + \dots + SS_{rc}$

**For each cell, all scores within that cell are subtracted from cell mean, squared, and summed**

$$SS_W = \sum_{j=1,1}^{rc} \left( \sum_{i=1}^{n_{rc}} (X_{irc} - \bar{X}_{rc})^2 \right)$$
$$SS_W = \sum_{i=1}^n X_i^2 - \frac{\left( \sum_{i=1}^n X_{cell11} \right)^2 + \left( \sum_{i=1}^n X_{cell12} \right)^2 + \dots + \left( \sum_{i=1}^n X_{cell rc} \right)^2}{n_{cell}}$$
$$SS_W = SS_T - SS_R - SS_C - SS_{RC}$$

# DEGREES OF FREEDOM

$$df_{Total} = N_T - 1$$

- Partitioned into 4 parts

- $df_{Total} = df_R + df_C + df_{RC} + df_W$ 
  - $df_R = r - 1$
  - $df_C = c - 1$
  - $df_{RC} = (r - 1)(c - 1)$
  - $df_W = (N - rc)$ 
    - **Assumes**  $n$  are same for all cells
    - Otherwise,  $\Sigma(n_{rc} - 1)$ : sum of  $n - 1$  per cell

# VARIANCE ESTIMATES

- Obtain 4 variance estimates when each variance component is divided by its *df*
  - $MS_R$  = Row variance estimate
    - Sensitive to effects of factor A
  - $MS_C$  = Column variance estimate
    - Sensitive to effects of factor B
  - $MS_{RC}$  = Row x Column variance estimate
    - Sensitive to interaction effects of A and B
  - $MS_W$  = Within-cells variance estimate
    - Not sensitive to effects of any factor

# *F*-STATISTICS

- Significance testing of 3 variance estimates
  - Distinct  $F_{stat}$  for each
    - $MS_R / MS_{Within}$  : Factor A
    - $MS_C / MS_{Within}$  : Factor B
    - $MS_{RC} / MS_{Within}$  : Interaction between factors A and B
- Each  $F_{stat}$  compared to distinct  $F_{crit}$ 
  - Based on  $df_{Effect}$  (e.g.,  $df_R$ ) and  $df_{Within}$
  - Reject  $H_0: F_{stat} > F_{crit}$



# SUMMARY TABLE

Source	<i>SS</i>	<i>df</i>	<i>MS</i>	<i>F</i>	<i>p</i>
<b>Row</b>					
<b>Column</b>					
<b>R x C</b>					
<b>Within</b>				<b>X</b>	<b>X</b>
<b>Total</b>			<b>X</b>	<b>X</b>	<b>X</b>

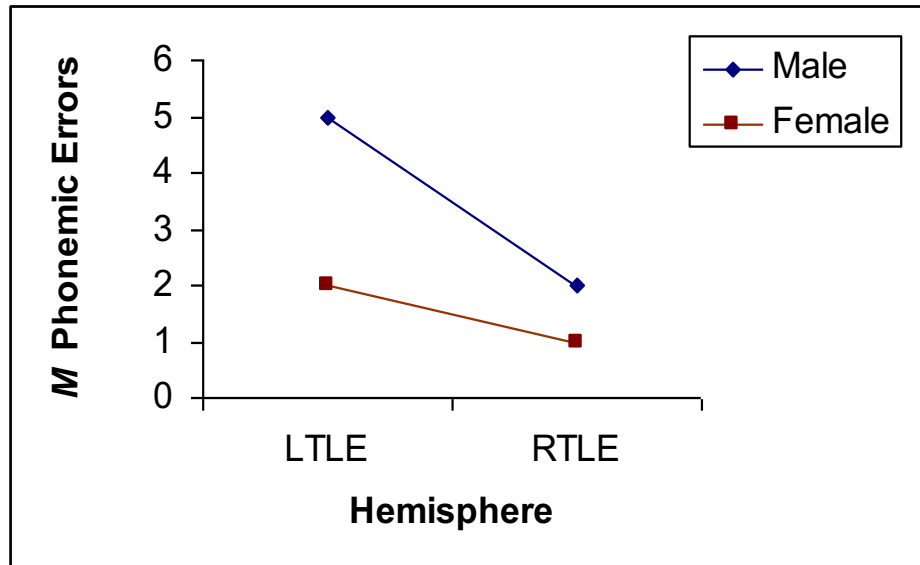
# INTERACTIONS

- Interaction between
  - 2 factors: 2-way interaction
  - 3 factors: 3-way interaction
    - Quite rare, be skeptical
- **Significance** indicates that the effect of 1 factor is not same at all levels of another factor
  - i.e. **the effect of 1 factor *depends* on the level of the other**
  - Effect of variables combined is different than would be predicted by either variable alone
- Most interesting results, but more difficult to explain or interpret than main effects

# INTERACTIONS

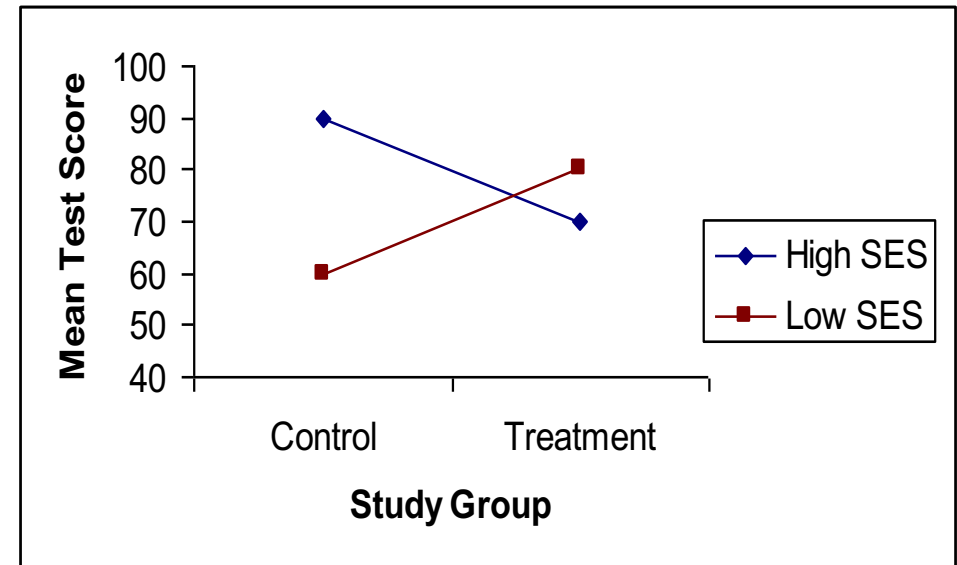
## Ordinal

- Direction or order of effects is similar for different subgroups



## Disordinal

- Direction or order of effects is reversed for different subgroups

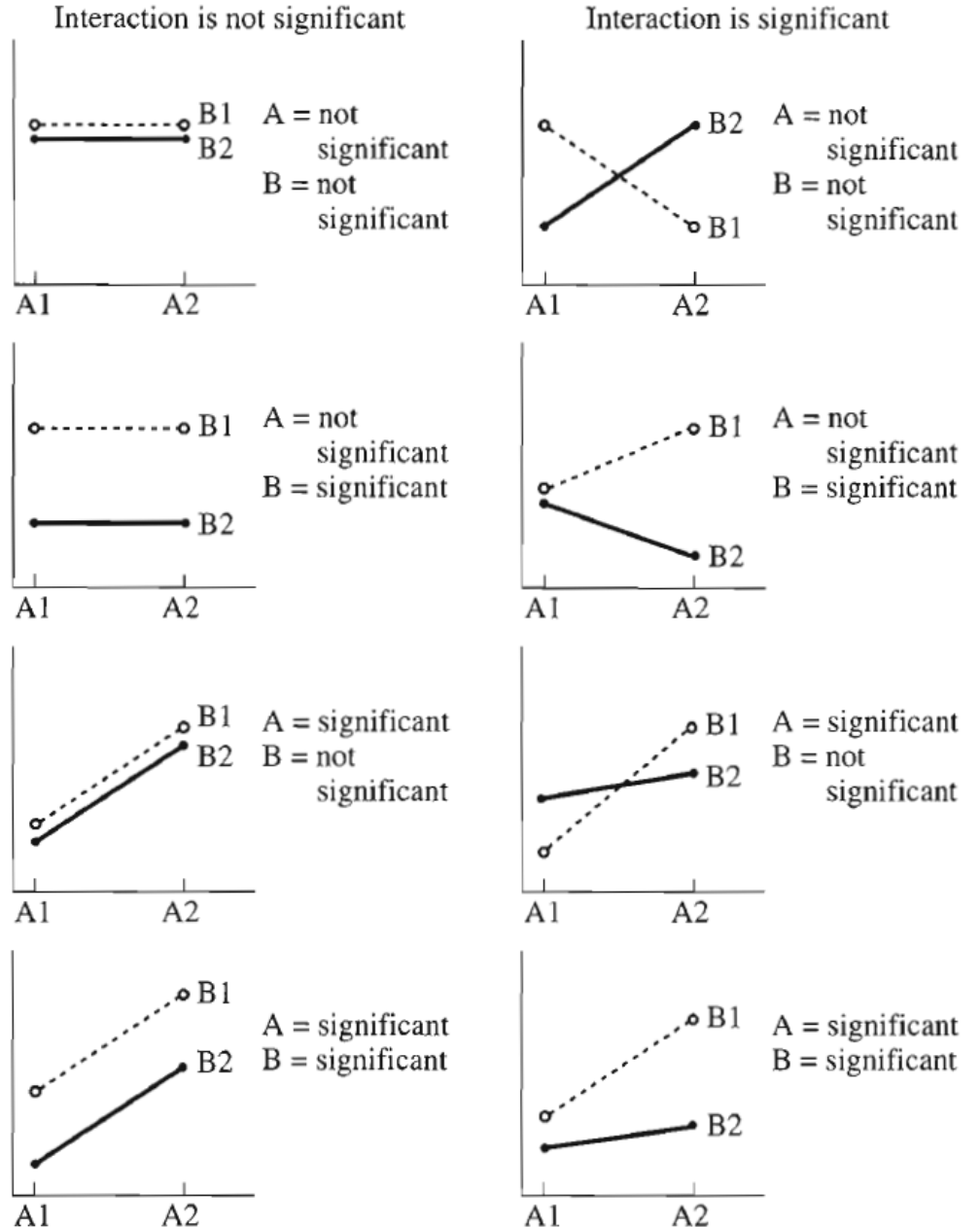


# INTERACTIONS

- Significance of interaction always evaluated 1<sup>st</sup>
  - If significant, interpret interaction, not main effects
  - If non-significant, interpret main effects
- Once we know effects of 1 factor are tempered by or contingent on levels of another factor (as in an interaction), interpretation of either factor (main effect) alone is problematic
- Best interpreted through visualization
  - Cell means plot
  - Interactions exist if lines cross or will cross (non-parallel)
- Design graph to best illustrate
  - Outcome on y-axis
  - Select factor for x-axis
  - Other factor(s) represented by separate lines
    - Selection guides interpretation, can dictate whether plot is ordinal/disordinal

A = Column means

B = Row means



# INTERACTIONS

- Some recommend only interpreting significant main effects (Keppel & Wickens, 2004) ...
  - *When there is no significant interaction*
  - (*Cautiously*) when there is a significant interaction, but 1) interaction effect size is small relative to that of main effects and 2) there is an ordinal pattern to the means
- However, must **report** all main and interaction effects regardless of statistical significance



# NEED FOR TESTING INTERACTIONS

- Results may be distorted if additional factors are not included in analysis so that interactions are not tested
  - E.g., If experimental effects of a drug had opposite effects in men and women, the variable representing drug effects may appear to be ineffective (non-significant main effect) without including the variable for sex differences
- If interaction terms are non-significant, increased confidence that effect of key factor (e.g., drug treatment) is generalizable to all levels of other factors (e.g., sex)

# EXAMPLE FROM TEXT

- Effect of sleep deprivation and compensating stimulation on performance of complex motor task
  - Outcome: Video game score simulating driving truck at night
  - Factor A (Row): Sleep deprivation
    1. Control: Normal sleep schedule
    2. Jet lag: Normal sleep amount, but during different hours
    3. Interrupted: Normal sleep amount, but only for 2 hours at a time
    4. Total Deprivation: No sleep for 4 days
  - Factor B (Column): Stimulation conditions
    1. Placebo: Told they are given a stimulant pill (really placebo)
    2. Caffeine: Told they are given a stimulant pill (really stimulant)
    3. Reward: Given mild electric shocks for mistakes and given a monetary reward for good performance

# EXAMPLE

- $H_0$  Deprivation

- $\mu_{control} = \mu_{jetlag} = \mu_{interrupted} = \mu_{deprive}$

- $H_0$  Stimulus

- $\mu_{placebo} = \mu_{caffeine} = \mu_{reward}$

- $H_0$  Interaction

- Effect of two factors is additive (no multiplicative or interaction effect)
  - Effect of 1 factor does NOT depend on level of other factor

# EXAMPLE

		Stimulus Type		
		Placebo	Caffeine	Reward
Deprivation Type	Control	24	26	28
		20	22	23
		29	20	24
		20	30	30
		28	27	33
	Jet Lag	22	25	26
		18	31	20
		16	24	32
		25	27	23
		27	21	30
	Interrupt	16	23	16
		20	28	13
		11	26	12
		19	17	18
		14	19	19
	Total Deprivation	14	23	15
		17	16	11
		12	26	19
		18	18	11
		10	24	17

Score	Stimulus	Stimulus	Deprivation	Deprivation
24	1	Placebo	1	Control
20	1	Placebo	1	Control
29	1	Placebo	1	Control
20	1	Placebo	1	Control
28	1	Placebo	1	Control
22	1	Placebo	2	Jet Lag
18	1	Placebo	2	Jet Lag
16	1	Placebo	2	Jet Lag
25	1	Placebo	2	Jet Lag
27	1	Placebo	2	Jet Lag
16	1	Placebo	3	Interrupt
20	1	Placebo	3	Interrupt
11	1	Placebo	3	Interrupt
19	1	Placebo	3	Interrupt
14	1	Placebo	3	Interrupt
14	1	Placebo	4	Total Dep
17	1	Placebo	4	Total Dep
12	1	Placebo	4	Total Dep
18	1	Placebo	4	Total Dep
10	1	Placebo	4	Total Dep
26	2	Caffeine	1	Control
22	2	Caffeine	1	Control
20	2	Caffeine	1	Control
30	2	Caffeine	1	Control
27	2	Caffeine	1	Control



# EFFECT SIZE



Proportion of variation in outcome accounted for by a particular factor or interaction term

- **Eta-squared ( $\eta^2$ )**

- **1-way ANOVA**

- $SS_{Between} / SS_{Total}$

- **2-way ANOVA**

- **Row factor:**  $SS_R / SS_{Total}$

- **Column factor:**  $SS_C / SS_{Total}$

- **Interaction:**  $SS_{RC} / SS_{Total}$

- **Interpretation:**

- **Range: 0 to 1**

- **Small:** .01 to .06

- **Medium:** .06 to .14

- **Large:** > .14

# EFFECT SIZE

- $\eta^2$  are biased parameter estimates
- Should estimate omega squared ( $\omega^2$ )
  - Substitute  $SS$  and  $df$  values

$$\omega^2 = \frac{SS_{Effect} - df_{Effect} * MS_{Within}}{SS_{Total} + MS_{Within}}$$

- Same interpretation as  $\eta^2$

# EFFECT SIZE

- When all factors are experimental or when many factors are included in analysis,  $SS$  due to a factor or interaction will be small relative to  $SS_{Total}$
- Partial effect size estimates are often reported
  - Proportion of variation in outcome accounted for by a particular factor or interaction term, excluding other main effects or interaction sources of variation

$$\text{Partial } \eta^2 = \frac{SS_{Effect}}{SS_{Effect} + SS_{Within}}$$

$$\text{Partial } \omega^2 = \frac{df_{Effect} (MS_{Effect} - MS_{Within}) / N}{[df_{Effect} (MS_{Effect} - MS_{Within}) / N] + MS_{Within}}$$

# MULTIPLE COMPARISONS

- Factorial ANOVA produces omnibus results
  - No indication of specific level (group) differences within or across factor(s)
- **Multiple comparisons** elucidate differences within significant main effects or interactions
- **Pattern of results dictates approach**
  - E.g., Significant main effects, but no interaction
- Each of the 3  $F$ -tests in a 2-Way ANOVA represents a ‘planned comparison’
  - No adjustment to  $\alpha_{EW}$  necessary
- However, within each main-effect and interaction a separate family of possible multiple comparisons may be conducted
  - $\alpha_{EW}$  *must be controlled within each ‘family’*



# NON-SIGNIFICANT INTERACTION

- Evaluation of significant main effect(s)
  - Factors with 2 levels
    - *No multiple comparisons required*
  - Factors with  $> 2$  levels
    - 2-way ANOVA is reduced to two 1-Way ANOVAs
    - *Simple (pairwise) or complex (linear) contrasts are computed within individual significant main-effect(s) (ignoring others)*

# NON-SIGNIFICANT INTERACTION

Significant main-effects

		B		Marginals
		B1	B2	
A	A1	$M_{11}$	$M_{12}$	$M_{A1}$ $M_{A2}$ $M_{A3}$
	A2	$M_{21}$	$M_{22}$	
	A3	$M_{31}$	$M_{32}$	
Marginals		$M_{B1}$	$M_{B2}$	

Simple or complex comparisons among marginal means (levels)

No further tests if  $F$ -test of main-effect indicates difference

# EXAMPLE 1: NON-SIGNIFICANT INTERACTION

- Sleep deprivation, stimulant, and motor performance example

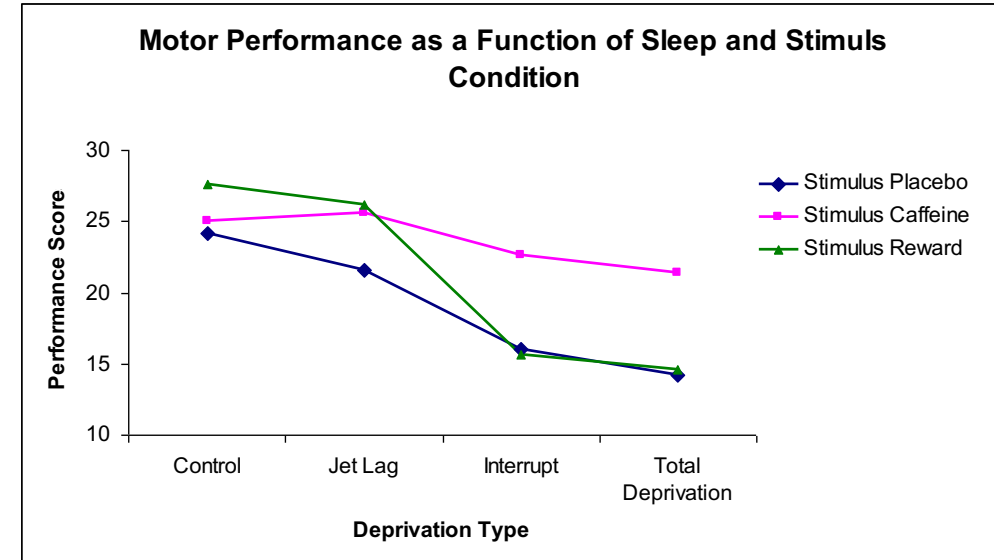
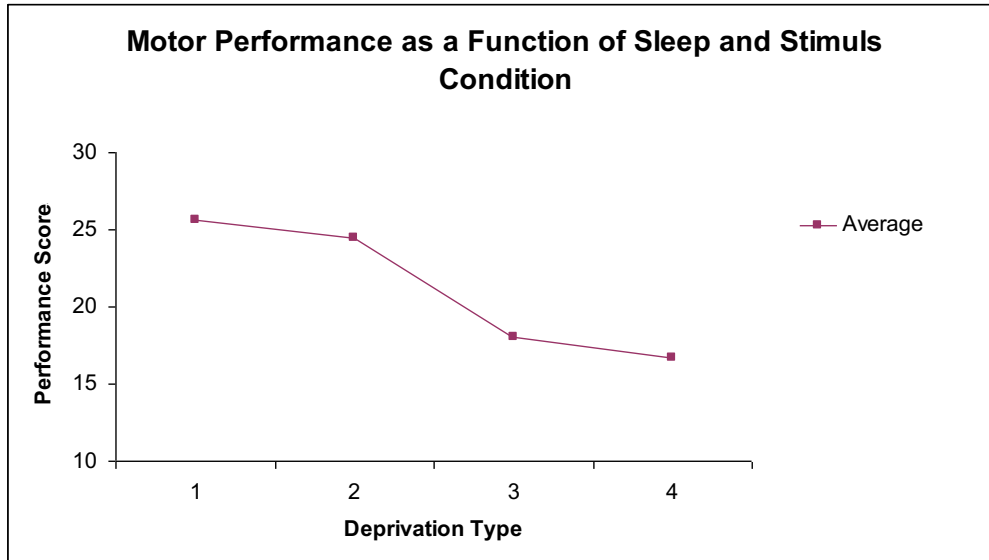
Anova Table (Type II tests)

Response: score

	Sum Sq	Df	F value	Pr(>F)	
Deprivation	897.0	3	18.2406	4.896e-08	***
Stimulus	217.6	2	6.6385	0.002849	**
Interaction	194.8	6	1.9803	0.087003	.
Residuals	786.8	48			

- Non-significant interaction
- Both main-effects are significant
- Need to compare 'marginal means' for differences among levels

# EXAMPLE 1: NON-SIGNIFICANT INTERACTION



- Figure on left indicates main effect for deprivation type collapsing across levels of stimulant type
  - 'Average' of simple (main) effects
  - Simple main effects are shown by the lines in figure on right
- When interaction is tested it is really a test of the  $H_0$  that all simple effects are similar

# EXAMPLE 1: NON-SIGNIFICANT INTERACTION

- Run 1-Way ANOVA on main-effects deemed significant in 2-Way ANOVA
  - Optional
- Run multiple comparisons, controlling  $\alpha_{EW}$  within each contrast
  - Pairwise: Tukey, Bonferroni
  - Linear contrasts: Contr.helmert

# EXAMPLE 1: NON-SIGNIFICANT INTERACTION

- Conduct 1-Way ANOVA in R as before, select pairwise comparisons for Tukey tests
- Alternative ‘by hand’;  $p$ -values close, not exactly the same

```
TukeyHSD(aov_4_object$aov, "dep_F", ordered = F)
```

```
TukeyHSD(aov_4_object$aov, "stim_F", ordered = F)
```

```
plot(TukeyHSD(aov_4_object$aov, "dep_f"))
```

```
plot(TukeyHSD(aov_4_object$aov, "stim_f"))
```

# SIGNIFICANT INTERACTION

- **Simple (main) effects** of interaction are tested
- **One factor is selected as stratifying factor**
  - Similar to deciding which factor to put on x-axis in means plot
  - Let theory and research questions guide selection
  - Levels (cells) of other factor are compared within each level of stratified factor
  - Can redo analysis by reversing which factor is stratified and which is examined
- *Comparing cell, rather than marginal, means*

# SIGNIFICANT INTERACTION

		B		Marginals
		B1	B2	
A	A1	$M_{11}$	$M_{12}$	$M_{A1}$
	A2	$M_{21}$	$M_{22}$	$M_{A2}$
	A3	$M_{31}$	$M_{32}$	$M_{A3}$
Marginals		$M_{B1}$	$M_{B2}$	

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		B1	B2	
A	A1	$M_{11}$	$M_{12}$	$M_{A1}$
	A2	$M_{21}$	$M_{22}$	$M_{A2}$
	A3	$M_{31}$	$M_{32}$	$M_{A3}$
Marginals		$M_{B1}$	$M_{B2}$	

- Simple main effects generally tested within each level of stratifying factor

- 2-levels

- Simple, pairwise comparisons: Tukey HSD or *t*-tests with Bonferroni correction

- > 2 levels

- Modified 1-way ANOVA followed by simple or complex comparisons



# SIGNIFICANT INTERACTION

- Modified 1-Way ANOVA tests of simple main effects often done 'by hand'
  - Obtain  $MS_{Between}$  from standard 1-Way ANOVA
    - Comparing means across 1 level of 1 factor within 1 level of another factor
  - Obtain  $MS_{Within}$  from original 2-Way ANOVA
    - Ensure homogeneity of variance assumption is reasonably satisfied

$$F_{Simple\ Effect} = \frac{MS_{Between\ (1\text{-way\ ANOVA)}}}{MS_{Within\ (2\text{-way\ ANOVA)}}$$

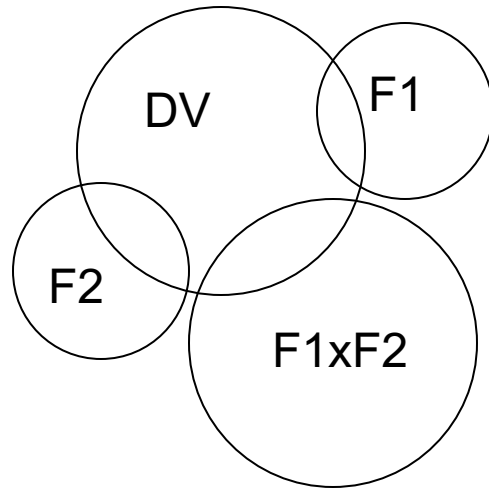
$F_{Critical}(df_{Between}, df_{Within})$  from 1-way ANOVA

# UNBALANCED DESIGNS

- Equal  $n$ s in each cell = Orthogonal design
  - Factors are independent/uncorrelated so that significance of any effect is independent of significance of other effects (including interaction)
- Most research consists of unbalanced data
  - As  $n$ s across cells become more unequal, factors become more dependent/correlated
    - Unbalanced:  $SS_{Between} \neq SS_R + SS_C + SS_{RC}$
  - More difficult to determine independent effects of each factor
- Previous equations and R commands will not work correctly for unbalanced designs

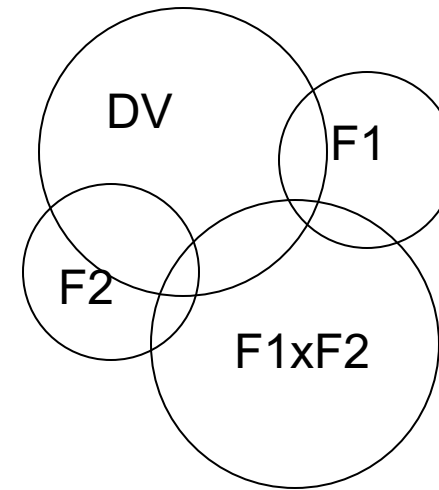
# UNBALANCED DESIGNS

- **Balanced**



- Sum of areas where factors overlap with DV =  $SS_B$ 
  - Remaining portion of DV =  $SS_W$

- **Unbalanced**



- Sum of areas where factors overlap with DV  $\neq SS_B$ 
  - Some areas counted twice
  - Remaining portion of DV =  $SS_W$

1. Equal cell sizes			
	Factor A		
Factor B	a1	a2	Row Marginal Means
b1	M = 100 n = 50	M = 150 n = 50	M = 125 n = 100
b2	M = 200 n = 50	M = 250 n = 50	M = 225 n = 100
Column Marginal Means	M = 150 n = 100	M = 200 n = 100	
2. Unequal cell sizes			
	Factor A		
Factor B	a1	a2	Row Marginal Means
b1	M = 100 n = 10	M = 150 n = 90	M = 145 n = 100
b2	M = 200 n = 90	M = 250 n = 10	M = 205 n = 100
Column Marginal Means	M = 190 n = 100	M = 160 n = 100	

Individual cell means and marginal *ns* are the same across both tables. Main effects (marginal means) differ across tables as a function of different cell *ns*. Conclusions from ANOVA may vastly differ.



# UNBALANCED DESIGNS

- Reason for unequal  $n$ s should be random, not related to factor(s) themselves (more difficult with non-experimental studies)
  - If not so, validity of results is questionable when regular ANOVA procedures are employed
- Adjustments made to ANOVA to correct for unequal  $n$ s
  - Analysis of weighted means: Non-recommended, but common, approach where imbalance is slight and imbalance is random
    - Harmonic mean of cell  $n$ s is used in computation of various  $MS$
    - Total  $N$  is adjusted = Harmonic mean of all cell sizes x # cells
    - $MS_{Within}$  = Weighted average of cell variances
    - Each row and column mean computed = Simple (non-weighted) average of cell means in a given row or column
  - Alternate  $SS$  calculations to handle overlapping variation accounted for in outcome (Coming up next!)
  - Regression analysis (Take EDUC/PSY 7610!)

$$\text{Harmonic mean} = \frac{\# \text{cells}}{\sum_{i=1}^n \frac{1}{n_i}}$$

# ALTERNATIVE *SS* CALCULATIONS

- Several methods for partitioning or allocating variation between outcome and factor(s) to account for unbalanced designs
  - Commonly used
    - Type I *SS*: Sequential or Hierarchical
    - Type II *SS*: Partially Sequential
    - Type III *SS*: Simultaneous or Regression
  - Specialized and less commonly used
    - Type IV *SS*: Don't use
    - Type V *SS*: Used for fractional factorial designs
    - Type VI *SS*: Effective hypothesis tests though sigma-restricted coding

# ALTERNATIVE SS RECOMMENDATIONS

- **Type II or III SS recommended in most cases**
  - Results should be fairly consistent
  - Type III is most commonly used
  - Nothing wrong with Type II
    - Considered by some to be more powerful, especially when testing main effects
    - Uncertainty of results when  $n$  are vastly unbalanced
- **Not an issue when design is balanced**
  - Type I-III yield same results
  - Even when unbalanced, interaction result same

# INTERACTION CONTRASTS

- An alternative is to perform ‘**interaction contrasts**’, rather than immediately testing simple effects
  - With a 2x2 design, only tests of simple main effects are possible
  - With a 2x3 design, 3 separate 2x2 ANOVAs may be conducted
    - Interaction magnitude (and significance) can differ from one subset to another
    - Simple effects can be used following significant interaction subsets
    - $MS_B$  for overall interaction = ‘average’ of  $MS_{Interactions}$  for separate interaction subsets



# IGNORING FACTORIAL DESIGN

- **Treat each cell as a separate group** (e.g., M/Rep, M/Dem, F/Rep, F/Dem) and run analysis as 1-Way ANOVA with R\*C groups?
  - Results in same  $SS_{Between}$  as factorial design ( $SS_R + SS_C + SS_{RC}$  ; when study is balanced)
  - Cannot see patterns in data, as all levels of all factors are blended together in each group
    - *Cannot as easily observe interaction effects*
  - Limits identification of characteristics that uniquely differentiate participants
  - More cumbersome when many factors included
  - Less powerful

# REPORTING RESULTS

- Marginal  $M$ s for main effects, cell  $M$ s for interactions and their  $SD$ s (or  $SE$ s) and  $CI$ s
- No need to report  $MS_W$
- For each significant effect
  - $F(df_{Effect}, df_{Within}) = F_{stat}, p\text{-value}, \text{effect size } (\eta^2 \text{ or } \omega^2)$
- Results of post-hoc or planned comparisons
- **Figures are \*extremely\* helpful!**

# CONCLUSIONS

- **With a non-significant interaction**
  - # of follow-up tests on main-effects needs to be kept low so as to not inflate  $\alpha_{EW}$ , where each main-effect can contain a family of tests
- **With a significant interaction**
  - # of tests of simple effects or interaction contrasts should not exceed  $df_{Interaction}$
  - For 2x2 ANOVA:  $\# \leq (r-1)*(c-1)$ 
    - In `Conformity` data example =  $2*1 = 2$  tests
  - Some forgo tests of simple main effects and compute all possible pairwise comparisons at cell level
  - Results in many, many tests
- **Following a significant interaction and significant simple main effects**
  - Not necessary to conduct all possible pairwise comparisons
  - Planned comparisons should be derived from theory or previous research and flow from research questions
- Significant unplanned interactions that do not conform to theory should be swallowed with a **HIGH DEGREE OF SKEPTICISM**